($2\,^{1}/_{2}$ Hours) **Total Marks : 60**

N.B. : (1) **All** questions are **compulsory**.

(2) **Figures** to the **right** indicate **full** marks.

(3) Draw **neat** diagrams wherever **necessary**.

(4) Symbols have usual meanings unless otherwise stated.

(5) Use of **non-programmable** calculator is allowed.

1. (a) Attempt any **one:---**

(i) Show that the change in the entropy due to mixing of two ideal gases results in to the paradox. Explain how the paradox is resolved. **8**

(ii) Consider two systems in contact with each other and isolated from surrounding. Using statistical concept of possible microstates accessible for the composite system, show that at equilibrium: $T_1 = T_2$, $P_1 = P_2$ and $\mu_1 = \mu_2$ **8**

(b) Attempt any **one:---**

(i) Describe the problem of one-dimensional simple harmonic oscillator. Hence show that the fundamental volume of the phase space is given by: $\omega_o = h$. **4**

(ii) Let the entropy of the classical ideal gas is given by **4**

$S(N,V,E) = Nk\,ln\left[\frac{V}{h^3}\left(\frac{4\pi mE}{3N}\right)^{3/2}\right] + \frac{3}{2}Nk$. Obtain equation for the specific heat at constant volume C_v and the specific heat at constant pressure C_p.

2. (a) Attempt any **one:---**

(i) State and prove Equipartition theorem. **8**

(ii) Consider a system of N quantum harmonic oscillators with frequency ω. Derive the expression for N-particle partition function and prove that the Helmholtz free energy for the system is **8**

$A = N\left[kT.\ln\left(1 - e^{-\beta\hbar\omega}\right) + \frac{1}{2}\hbar\omega\right]$.

(b) Attempt any **one**:---

(i) Show that the energy fluctuations ΔE in canonical ensemble follows: **4**

$\langle(\Delta E)^2\rangle = kT^2 C_v$

(ii) Show that the N-particle partition function for a classical ideal gas is given **4**

by $Q_N(V,T) = \frac{1}{N!}\left[\frac{V}{h^3}(2\pi mkT)^{3/2}\right]^N$

Use $\int_0^\infty dx.x^2.e^{-ax^2} = \frac{1}{4a}\sqrt{\frac{\pi}{a}}$.

3. (a) Attempt any **one**:---

(i) Derive the relation for density fluctuation in grand canonical ensemble given by **8**

$$\frac{\overline{(\Delta n)^2}}{\bar{n}^2} = \frac{kT}{V}\kappa_T$$

where κ_T is isothermal compressibility of the system.

(ii) What do you mean by phase equilibrium? Show that Gibb's free energy is minimized at equilibrium. Hence derive the Clausius-Clapeyron equation. **8**

(b) Attempt any **one**:---

(i) Sketch P-T phase diagram for Helium-4. State its properties in different phases. **4**

(ii) Partition function of a system of independent localized particles is given by **4**

$$Q_N(V,T) = [Q_1(V,T)]^N$$

Obtain the expression for pressure P and number of particles N.

4. (a) Attempt any **one**:---

(i) For a linear harmonic oscillator, show that **8**

$$Tr\left(e^{-\beta\hat{H}}\right) = \frac{e^{-\frac{\beta\hbar\omega}{2}}}{1 - e^{-\beta\hbar\omega}}$$

(ii) Derive an expression for antisymmetric wave function of indistinguishable particles. **8**

(b) Attempt any **one:**---

(i) Write a short note on wave functions of indistinguishable particles. **4**

(ii) State and explain the postulate of equal a priori probabilities. **4**

5. Attempt any **four:**---

(a) Show that for micro-canonical ensemble: **3**

$P = -\left(\frac{\partial E}{\partial V}\right)_{N,S}$ and $\mu = \left(\frac{\partial E}{\partial N}\right)_{V,S}$

(b) Explain the phase space of a classical system. Hence discuss the concept of stationary ensemble. **3**

(c) For a system in canonical ensemble, show that $C_v = -T\left(\frac{\partial^2 A}{\partial T^2}\right)$. **3**

(d) What is the 'virial' for a system? State virial theorem. **3**

(e) Define fugacity z of the system. Show q-potential is logarithm of the grand canonical partition function. **3**

(f) Calculate the slope of the solid-liquid transition line for water near the triple point T = 273.16K, given that the latent heat of melting is 80cal/g, the density of the liquid phase is 1.00g/cm^3, and the density of the ice phase is 0.92g/cm^3. Estimate the melting temperature at P = 100atm. **3**

(g) What are the Hamiltonian and the wavefunction of a free particle in a three dimensional box of length L? **3**

(h) Write an expression for mean thermal wavelength. Explain each term. **3**
