($2\,{}^{1}/_{2}$ Hours) **[Total Marks : 60**

N.B. : (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate **full** marks.
(3) Draw **neat** diagrams wherever **necessary**.
(4) Symbols have usual meanings unless otherwise stated.
(5) Use of **non-programmable** calculator is allowed.

1. (a) Attempt any **one:---**

(i) Starting with the 4-force $f_\mu = \frac{1}{c} F_{\mu\nu} j_\nu$ show that f_μ can be written as $f_\mu = \partial_\nu T_{\mu\nu}$ where $T_{\mu\nu} = \frac{1}{4\pi}(F_{\mu\lambda}F_{\lambda\nu} + \frac{1}{4}\delta_{\mu\nu}F_{\alpha\beta}F_{\alpha\beta})$. **8**

(ii) State and prove Poynting theorem. Explain the terms Poynting vector and momentum density. **8**

(b) Attempt any **one:---**

(i) Express the components of the electric and magnetic fields in terms of the scalar and vector potentials. Define the field tensor in terms of the scalar and vector potentials. Express the field tensor in terms of the electric and magnetic fields. **4**

(ii) Show that $F_{\mu\nu}F_{\mu\nu}$ and $\epsilon_{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}$ are invariant under a Lorentz transformation. (note: You are NOT being asked to find the expressions in terms of the electric and magnetic fields) **4**

2. (a) Attempt any **one:---**

(i) Derive an expression for frequency dependence of conductivity. What is plasma frequency? Show that under the limit $\omega\tau \ll 1$ the conductivity is constant and is equal to σ_0. **8**

(ii) Explain the terms phase and group velocity. For the superposition of two waves of equal amplitudes and distinct but neighboring frequencies $\omega_1 k_1$ and $\omega_2 k_2$
$U(x,t) = A\left(e^{i(k_1 x - \omega_1 t)} + e^{i(k_2 x - \omega_2 t)}\right)$; obtain the expressions for group velocity and phase velocity. **8**

(b) Attempt any **one:---**

(i) For plane harmonic waves in matter obtain the relations, **4**
$k\text{x}H = -\frac{\omega}{c}\eta E$ and $\hat{k} \times E = \frac{\omega}{c}\mu H$

(ii) Discuss the classifications of fields in wave guides **4**

3. (a) Attempt any **one:---**

(i) The LW electric field for a point charge is given by: **8**

$$\vec{E} = e\left[\frac{(\hat{n}-\vec{\beta})(1-\beta^2)}{k^3R^2} + \frac{\hat{n}\times(\hat{n}-\vec{\beta})\times\vec{a}}{c^2k^3R}\right]$$

Using this relation show that the power P radiated, per unit solid angle for a collection of charges under non relativistic limit, having electric dipole moment $\vec{p}$ is given by

$$\frac{dP}{d\Omega} = \frac{(\ddot{\vec{p}})^2 \sin^2\theta}{4\pi c^3}$$

(ii) The retarded scalar potential is given by **8**

$$\phi(\vec{r},t) = \int \frac{\rho(\vec{r}',t_r)}{r} d\tau'$$

Show that it satisfies the equation

$$\vec{\nabla}^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -4\pi\rho$$

(b) Attempt any **one:---**

(i) Show that the Lorentz force law takes the form **4**

$$\frac{d\vec{P}}{dt} = -\vec{\nabla}U$$

Where $\vec{P} = \vec{p} + \frac{q}{c}\vec{A}$ and $U = q\left(\phi - \frac{\vec{v}}{c}\cdot\vec{A}\right)$ with $\vec{p}$ is linear momentum and ϕ, and $\vec{A}$ are scalar and vector potential respectively.

(ii) The LW Potentials are **4**

$$\phi(\vec{r},t) = \frac{qc}{[Rc - \vec{R}\cdot\vec{v}]}$$

$$\vec{A}(\vec{r},t) = \frac{q[\vec{v}]}{[Rc - \vec{R}\cdot\vec{v}]}$$

Where the square bracket indicates that the quantities are measured at retarded time. If $[\vec{v}]$ is constant, show that

$$\phi(\vec{r},t) = \frac{qc}{\sqrt{(c^2t - \vec{r}\cdot\vec{v})^2 + (r^2 - c^2t^2)(c^2 - v^2)}}$$

$$\vec{A}(\vec{r},t) = \frac{1}{c}\vec{v}\phi$$

4. (a) Attempt any **one:---**

(i) Write the expression for field tensor {F} .Form energy momentum tensor {T}.Obtain the term T_{jk}. What does the term T_{jk} represent? **8**

(ii) Write the Lagrangian for a non relativistic charge particle in electromagnetic field. Write the same for relativistic particle. Express it in terms of four velocity and four vector potential and apply Hamilton's Principle. **8**

(b) Attempt any **one**:---

(i) Obtain the continuity equation from the equation $\frac{\partial F_{\mu\nu}}{\partial x_{\nu}} = \frac{4\pi}{c} J_{\mu}$ **4**

(ii) For a charge particle in external electromagnetic field the Lagrangian function is **4**

$$L(x_{\mu}, \omega_{\mu}, \tau) = -m_0 c\sqrt{-\hat{\omega}^2} + \frac{e}{c} A_{\mu}\omega_{\mu},$$

Obtain Lagrange equation.

5. Attempt any **four**:---

(a) For a uniform wire of radius 'a' carrying dc current 'I' Determine the Poynting vector. **3**

(b) Show that the 4-volume element defined by $d^4x \equiv dx_1 dx_2 dx_3 dx_0$ where $x_0 \equiv x_4/i$ is an invariant under proper Lorentz transformation. **3**

(c) Show that for insulators anomalous dispersion is disregarded. **3**

(d) Write Maxwell's equation in vacuum with ρ=0 and j=0 and obtain the wave equation. **3**

(e) Suppose we have two sets of potentials, $(\phi, \vec{A})$ and $(\phi', \vec{A}')$ which corresponds to same electric and magnetic fields. By how much can they differ? **3**

(f) What do you understand by the term retarded potential? **3**

(g) Comment on the symmetry property of energy momentum tensor {T}. What is tr{T} =? **3**

(h) Write down the components of ∂_{α} **3**

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