

(Time: 2 1/2 Hours)

[Total Marks : 60

- N.B. :** (1) All questions are compulsory.
 (2) Figures to the right indicate full marks.
 (3) Draw neat diagrams wherever necessary.
 (4) Symbols have usual meanings unless otherwise stated.

1. (a) Attempt any one:---

08

- (i) What is Poynting vector S ? Obtain the expression for Poynting theorem Explain which physical quantity is expressed by S/c^2 .
 (ii) Write the x, y and z components of the following equations

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}; \vec{B} = \vec{\nabla} \times \vec{A} \text{ hence define field tensor and express it as a matrix.}$$

(b) Attempt any one:---

04

- (i) Express the law of conservation of linear momentum in electrodynamics in terms of Maxwellian stress tensor.
 (ii) Explain why the tail of a comet is always directed away from the Sun.

2. (a) Attempt any one:---

08

- (i) Using Maxwell's equations in vacuum obtain the wave equations for B and E . Write down the solutions of these equations and show that for electromagnetic waves travelling in vacuum, the amplitudes of the electric and magnetic vectors are the same.
 (ii) Obtain expression $\sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau}$ for frequency dependence of the conductivity. Discuss two cases for frequency dependence of the conductivity.

(b) Attempt any one:---

04

- (i) If electric field corresponding to certain plane monochromatic wave is

$$\vec{E}(r, t) = \hat{i}E_1 e^{i(k \cdot r - \omega t)} + \hat{j}E_1 e^{i(k \cdot r - \omega t + \pi/2)}$$

Then comment on the state of polarization.

- (ii) Write expressions for the real and imaginary parts of atomic polarizability α and explain normal and anomalous dispersion with the help of neat diagram.

3. (a) Attempt any **one**:---

08

- (i) Obtain an expression for the electric field and magnetic field produced by a charged particle in uniform motion, hence draw and explain polar plot of electric field magnitude.

Show that a total power radiated from a non-relativistic accelerated charge is

- (ii) given by

$$P = \frac{2e^2 a^2}{3c^3}$$

(b) Attempt any **one**:---

04

- (i) Show that the retarded potentials

$$\Phi(\vec{r}, t) = \iiint \frac{\rho(\vec{r}', t - \frac{R}{c})}{R} dv' \text{ and } \vec{A}(\vec{r}, t) = \frac{1}{c} \iiint \frac{\vec{J}(\vec{r}', t - \frac{R}{c})}{R} dv'$$

leads to the generalized Coulomb-Faraday law

$$\vec{E}(\vec{r}, t) = \iiint \left(\frac{[\rho] \mathbf{e}_R}{R^2} + \frac{[\partial \rho / \partial t] \mathbf{e}_R}{cR} - \frac{[\partial \vec{J} / \partial t]}{c^2 R} \right) dv'$$

where $\vec{R} = |\vec{r} - \vec{r}'|$ and \mathbf{e}_R is a unit vector along \vec{R} .

- (ii) Write down Maxwell-Ampere's equation in terms of scalar potential $\Phi(\vec{r}, t)$ and vector potential $\vec{A}(\vec{r}, t)$, substitute Lorentz' gauge condition in it and obtain the resulting equation.

4. (a) Attempt any **one**:---

08

- (i) Construct energy momentum tensor $T_{\mu\nu}$ and get the energy conservation law from it.
- (ii) Write the Lagrangian for relativistic charge particle in electromagnetic field and obtain the equation of motion.

(b) Attempt any **one**:---

04

- (i) Write the components of four vector current density J_μ , hence write the continuity equation in four vector notation.
- (ii) Express Lorentz gauge condition in four vector notations.

5. Attempt any four:---

12

- (i) The radiant power of the Sun is 3.8×10^{26} watts. Find the radiation pressure on a completely absorbing area near the Earth. (The distance between the Earth and the Sun $= 1.5 \times 10^{11} \text{ m}$, $c = 3 \times 10^8 \text{ m/s}$)
- (ii) Write the Maxwell equations in differential form.
- (iii) Replacing vector operator ∇ by $i\vec{k}$ and $\partial/\partial t$ by $-i\omega$, write modified Maxwell equations in matter
- (iv) What is penetration depth? Calculate penetration depth for light wave frequency $f = 4 \times 10^{10} \text{ Hz}$ and $P_2 = 1.3$
- (v) State the advantages of writing Maxwell equations in terms of scalar potential ϕ and vector potential A
- (vi) In the case of charged particle with collinear velocity and acceleration, draw polar plot and explain it.
- (vii) Explain the physical meaning of the equation $E^2 - B^2 = E'^2 - B'^2$
- (viii) Write down the components of ∂_α
