(2½ Hours) **[Total Marks :60**

N.B. : (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate **full** marks.
(3) **Symbols** have their usual **meaning** unless otherwise **stated**.
(4) Use of **log tables** and **non-programmable** calculator is **allowed**.

1. (a) Attempt any **one:---** **8**

(i) Explain time independent perturbation theory for non-degenerate states. Obtain the first-order perturbation corrections to the energy eigenvalues and eigenfunctions.

(ii) Consider an isotropic harmonic oscillator in two dimensions. The Hamiltonian is given by

$$H_0 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2}(x^2 + y^2)$$

A. What are the energies of the two lowest -lying states? Is there any degeneracy?

B. A perturbation, $H' = \varepsilon xy$, $(\varepsilon \ll 1)$ is applied on the system. Find the first-order correction to the ground and first excited states.

Hint: $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a}_x + \hat{a}_x^\dagger)$ $\hat{y} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a}_y + \hat{a}_y^\dagger)$

(b) Attempt any **one :---** **4**

(i) Explain Fermi's Golden rule.

(ii) The energy eigenfunctions for the infinite square well of width *a* is:

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right).$$

Find the first order correction to the energies and eigenfunctions for the perturbation:

$$H' = V_0, \qquad 0 < x < \frac{a}{2}$$
$$= 0, \qquad \frac{a}{2} < x < a$$

2. (a) Attempt any **one :---** **8**

(i) Use the variational method to estimate the ground state energy of a particle of mass *m* in the potential given by

$$V(x) = \infty, \qquad x \le 0$$
$$= \frac{1}{2}m\omega^2x^2, \quad x > 0$$

Take $\psi(x) = Axe^{-\alpha x^2}$ as the trial wave function where α is the variational parameter and A is the normalization constant.

Given: $\int_0^\infty x^{2n}e^{-\beta x^2}dx = \frac{(2n)!\sqrt{\pi}}{2^{2n+1}\, n!\, \beta^{(2n+1)/2}}$

(ii) Use WKB approximation to find the tunneling probability through a potential barrier.

(b) Attempt any **one** :--- **4**

(i) Show that variational method gives the upper bound of the ground state energy.

(ii) Obtain energy eigenvalues of harmonic oscillator using WKB approximation.

3. (a) Attempt any **one** :--- **8**

(i) A particle with mass m_1 is scattered elastically by a particle of mass m_2 at rest in the Lab frame.

A) Find the relation between the scattering angles of m_1 in Lab frame and the Centre of mass frame.

B) Find the relation between differential scattering cross-section in Lab and centre of mass frame.

(ii) Calculate the differential cross-section in the Born approximation for the potential $V(r) = \frac{V_0}{r} e^{-(r/a)}$. Also calculate the total cross-section.

(b) Attempt any **one** :--- **4**

(i) Calculate the total cross-section for low energy (S-wave) scattering of a particle of mass m from the following potential

$$V(r) = -V_{0,} \qquad r < r_0$$
$$= 0, \qquad r > r_0$$

(ii) Discuss the validity conditions of Born approximation.

4. (a) Attempt any **one** :--- **8**

(i) Consider a system of three noninteracting particles confined in a one-dimensional infinite potential well of length a. Determine the energy and wavefunction of the ground state and first excited state when the particles are

A) Spinless distinguishable with masses $m_1 < m_2 < m_3$

B) Identical Bosons

(ii) Obtain the plane wave solution for the spin half particle in the relativistic formalism. Write the wavefunctions corresponding to positive and negative energies and two spin states.

(b) Attempt any **one**:--- **4**

(i) What are negative energy states? What is a hole?

(ii) Obtain the equation of continuity from the Klein- Gordon equation.

5. Attempt any **four** :--- **12**

(a) A particle is initially in its ground state in a one-dimensional harmonic potential. A perturbation, $H' = V_0 \hat{x} e^{-t/\tau}$ is turned on at $t = 0$. Calculate the probability that the particle will be found in its first excited state after a sufficiently long time $(t \to \infty)$.

(b) A hydrogen atom is in a constant uniform electric field 'E' that points in the z direction. Calculate the first order correction to the ground state energy of the atom.

Given: Unperturbed ground state of hydrogen atom is $\phi_{100}(r) = \frac{1}{\sqrt{\pi}a_0^{3/2}} e^{-\left(r/a_0\right)}$

(c) Use WKB approximation to estimate the transmission coefficient of a particle of mass m and energy E, $(E < V_0)$ moving through the following potential barrier

$$V(x) = 0, \qquad x < 0$$
$$= V_0 - \lambda x, \qquad x > 0$$

(d) Discuss the validity condition of WKB approximation.

(e) What is scattering amplitude? How is it related to scattering cross section?

(f) Explain optical theorem.

(g) Obtain Klein-Gordon equation from relativistic energy relation.

(h) Show that: $\alpha_x \alpha_y = i\sigma'_z$

where α's are Dirac matrices.
