(2½ Hours) [Total Marks :60

N.B. : (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate **full** marks.
(3) **Symbols** have usual meaning unless otherwise **stated**.
(4) Use of **non-programmable** calculator is allowed.

1. (a) Attempt any **one:---** **8**

(i) What are Clebsch-Gordan coefficients?
Calculate the Clebsch-Gordon coefficients for addition of angular momenta $\vec{J}_1 = 1/2$ and $\vec{J}_2 = 1/2$.

(ii) $|s, m_s\rangle$ is the simultaneous eigenstates of $\hat{S}^2$ and $\hat{S}_z$. Find the matrix representation of operators $\hat{S}_x$. $\hat{S}_y$ and $\hat{S}_z$ in this basis.

(b) Attempt any **one:---** **4**

(i) Find the matrix representations of $\hat{J}^2, \hat{J}_z$ and $\hat{J}_x$ for $j = 1$ in the $|j, m\rangle$ basis.

(ii) A spin-1/2 particle is in the state

$$\chi = A\begin{pmatrix} 1-2i \\ 2 \end{pmatrix}$$

A) Determine *A*.
B) What are the probabilities of getting $+\hbar/2$ and $-\hbar/2$ if you measure S_z?

2. (a) Attempt any **one:---** **8**

(i) Explain time independent perturbation theory for non-degenerate states. Obtain the first order perturbation correction to the eigenvalues and eigenfunctions.

(ii) A system described by a Hamiltonian $\hat{H}_0(\vec{r})$ with known solution set $\{E_n.\psi_n(\vec{r},t)\}$. Initially $(t \to -\infty)$ the system in the state $\psi_l(\vec{r},t)$ subjected to a time dependent perturbation $\hat{H}'(\vec{r},t) = V(\vec{r})f(t)$. Obtain the probability of transition of the system from the state $\psi_l(\vec{r},t)$ to $\psi_k(\vec{r},t)$ in time t.

(b) Attempt any **one:---** **4**

(i) The energy eigenfunctions for the infinite square well of width *a* is:

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right).$$

Find the first order correction to the energies and eigenfunctions for the perturbation:

$$H' = V_0, \qquad 0 < x < \frac{a}{2}$$
$$= 0, \qquad \frac{a}{2} < x < a$$

(ii) Explain Fermi's Golden rule.

3. (a) Attempt any **one**:--- **8**

(i) Use the variational method to estimate the ground state energy of a particle of mass m in the potential given by

$$V(x) = \infty, \quad x \leq 0$$
$$= \frac{1}{2}m\omega^2x^2, \quad x > 0$$

Take $\psi(x) = Axe^{-\alpha x^2}$ as the trial wave function where α is the variational parameter and A is the normalization constant.

Given: $\int_0^\infty x^{2n}e^{-\beta x^2}dx = \frac{(2n)!\sqrt{\pi}}{2^{2n+1}\, n!\, \beta^{(2n+1)/2}}$

(ii) Use WKB approximation to find the tunneling probability through a potential barrier.

(b) Attempt any **one**:--- **4**

(i) Show that variational method gives the upper bound of the ground state energy.

(ii) Obtain energy eigenvalues of harmonic oscillator using WKB approximation.

4. (a) Attempt any **one**:--- **8**

(i) Derive the expression of scattering amplitude using partial wave analysis.

(ii) Calculate the differential cross-section in the Born approximation for the potential $V(r) = \frac{V_0}{r}e^{-(r/a)}$. Also calculate the total cross-section.

(b) Attempt any **one**:--- **4**

(i) Calculate the total cross-section for low energy (S-wave) scattering of a particle of mass m from the following potential

$$V(r) = -V_0, \quad r < r_0$$
$$= 0, \quad r > r_0$$

(ii) Discuss the validity conditions of Born approximation.

5. Attempt any **four**:--- **12**

(a) Show that the components of Pauli spin matrices $\hat{\sigma}$ anticommute.

(b) Calculate: A) $[\hat{J}_x^2, \hat{J}_y]$ B) $[\hat{J}^2, \hat{J}_y]$

(c) A hydrogen atom is in a constant uniform electric field 'E' that points in the z direction. Calculate the first order correction to the ground state energy of the atom.

Given: Unperturbed ground state of hydrogen atom is $\phi_{100}(r) = \frac{1}{\sqrt{\pi}a_0^{3/2}}e^{-(r/a_0)}$

(d) A particle is initially in its ground state in a one-dimensional harmonic potential. A perturbation, $H' = V_0 \hat{x} e^{-t/\tau}$ is turned on at $t = 0$. Calculate the probability that the particle will be found in its first excited state after a sufficiently long time $(t \to \infty)$.

(e) Use WKB approximation to estimate the transmission coefficient of a particle of mass m and energy E, $(E < V_0)$ moving through the following potential barrier

$$V(x) = 0, \qquad x < 0$$
$$= V_0 - \lambda x, \qquad x > 0$$

(f) Explain briefly the Rayleigh Ritz method using a linear combination of fixed basis function ψ_i as trial wave function and treat the expansion coefficient as variable parameter.

(g) Explain optical theorem.

(h) What is scattering amplitude? How is it related to scattering cross section?
