Time : 2 Hrs **Marks : 50**

N.B. : (1) **All** questions are **compulsory**.

(2) **Figures** to the **right** indicate **full** marks.

(3) Draw **neat** diagrams wherever **necessary**

(4) Symbols have usual meanings unless otherwise stated.

(5) Use of **non-programmable** calculator is allowed.

1. **(a)** Attempt any **one:-**

(i) In a double slit experiment, prove that when two non-interacting beams with intensity I_1 and I_2 combine in same region of space, the resultant intensity is $I = I_1 + I_2$, if beam is composed of particle, while resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\alpha_1 - \alpha_2)$, if beam is composed of waves. Here $(\alpha_1 - \alpha_2)$ is phase difference between two beams. Interpret the result. **7**

(ii) Wavefunction for a system of particle confined to a region $x \in [0, L]$ is given by **7**

$$\psi(x) = \alpha \sin\left(\frac{3\pi x}{L}\right)$$

(a) Find value of normalization constant α in the wavefunction.

(b) Calculate probability of finding particle in range $\frac{L}{3}$ to $\frac{2L}{3}$.

(c) Assume particle behaves like free particle inside the region $x \in [0, L]$ and show that energy eigenvalue for particle is $9\frac{\hbar^2\pi^2}{2mL^2}$.

(b) Attempt any **one:-**

(i) Evaluate following commutator relations: **3**

(a) $[\hat{p}_x, \hat{y}]$
(b) $[\hat{p}_x, \hat{x}^2]$

(ii) Find linear momentum expectation value for following wavefunction **3**

$$\psi(x) = \frac{1}{\sqrt{a}}\, e^{ikx}, \qquad for\ x \in [0, a]$$

2. (a) Attempt any **one:-**

(i) (a) Check if the following operators are Hermitian **7**

(1) $(\hat{A}+\hat{A}^{\dagger})$, (2) $i(\hat{A}+\hat{A}^{\dagger})$

(b) For Hermitian operator, prove that all of its eigenvalues are real and the eigenvectors corresponding to different eigenvalues are orthogonal.

(ii) (a) Write a note on Schrodinger Picture **7**

(b) Consider two states $|\psi\rangle = i|\phi_1\rangle + 3i|\phi_2\rangle - |\phi_3\rangle$ where $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_3\rangle$ are orthonormal, calculate $\langle\psi|\psi\rangle$.

(b) Attempt any **one:-**

(i) How operators transform under unitary transformation? Show that if operator $\hat{A}$ is Hermitian then its transform $\hat{A}'$ is also Hermitian. **3**

(ii) State any 3 properties of Hilbert space. **3**

3. (a) Attempt any **one:-**

(i) Show that the energy and total momentum of an isolated system are constants of the motion. **7**

(ii) Derive an expression for one dimensional harmonic oscillator and show in which domain the wave function is (a) Oscillatory (b) Non-Oscillatory. **7**

(b) Attempt any **one:-**

(i) Show that in the nth Eigen state of the harmonic oscillator, the average kinetic energy <T> is equal to the potential energy <V>. **3**

(ii) Show that T+R=1 for all one-dimensional barrier problems. **3**

4. (a) Attempt any **one:-**

(i) Express operator form of L_Z in spherical polar coordinates. **7**

(ii) The Schrodinger equation for hydrogen atom can be defined as $\frac{d^2u}{d\rho^2} - \frac{l(l+1)}{\rho^2}u + \left(\frac{\lambda}{\rho} - \frac{1}{4}\right)u = 0$. Solve this equation when (a) ρ is very large i.e. $\rho \to \infty$ and (b) ρ is in neighborhood of origin i.e. $\rho \to 0$. **7**

Where $u = rR,\ \rho = 2kr,\ \lambda = \left(\frac{Ze^2}{\hbar} \cdot \sqrt{\frac{\mu}{2|E|}}\right)$.

(b) Attempt any **one:-**

(i) Show that $[L_x, L_y] = i\hbar L_Z$ **3**

(ii) Ground state of hydrogen atom is given by $\Phi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$. Find the value of r for which radial probability density is maximum. **3**

5. Attempt any **five:-**

(a) Explain the concept of de Broglie wavelength. **2**

(b) What are observables? Give 2 examples of observables. **2**

(c) Consider a Matrix A which represents operator $\hat{A}$, a ket $|\psi\rangle$ and a bra $\langle\phi|$: **2**

$$A=\begin{bmatrix} 5 & 3+2i & 3i \\ -i & 3i & 8 \\ 1-i & 1 & 4 \end{bmatrix}, |\psi\rangle = \begin{bmatrix} -1+i \\ 3 \\ 2+3i \end{bmatrix}, \langle\phi| = \begin{bmatrix} 6 & -i & 5 \end{bmatrix}$$

Calculate $\langle\phi|\hat{A}|\psi\rangle$

(d) Define Hermitian operator and state its properties. **2**

(e) Write down the Schrodinger equation for free particle of mass *m* and show the kinetic energy of the particle is $\frac{h^2k^2}{8\pi^2 m}$ **2**

(f) Under what conditions is the expectation of an operator ***A*** is constant in time? **2**

(g) Evaluate the minimum value of $\Delta L_y \Delta L_z$ **2**

(h) Evaluate $[L_Z, L_+]$ **2**

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