(2 Hours) **Total Marks: 50**

N.B. : (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate **full** marks.
(3) Draw **neat** diagrams wherever **necessary**
(4) Symbols have usual meanings unless otherwise stated.
(5) Use of **non-programmable** calculator is allowed.

1. (a) Attempt any **one:-**

(i) Evaluate the contour integral where c is the circle $|z| = 10$. **7**

$$\int_c \frac{z^2 - 2z}{(z+1)^2(z^2+4)} dz$$

(ii) Obtain the Taylor or Laurent series which represents the function **7**

$f(z) = \frac{1}{(z^2+1)(z+2)}$ when (i) $1 < |z| < 2$ (ii) $|z| > 2$

(b) Attempt any **one:-**

(i) Find the imaginary part of the analytic function whose real part is **3**

$x^3 - 3xy^2 + 3x^2 - 3y^2$

(ii) Show that the function $e^x(\cos y + i \sin y)$ is an analytic function, find its derivative. **3**

2. (a) Attempt any **one:-**

(i) Determine the eigenvalues and eigenvector of **7**

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

(ii) The Pauli spin matrices in quantum mechanics are **7**

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Show that ${\sigma_x}^2 = {\sigma_y}^2 = {\sigma_z}^2$ = a unit matrix.

Also show that any two of these matrices anti-commute.

(b) Attempt any **one:-**

(i) Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ *is* unitary. **3**

(ii) Using Levi-Civita symbol, show that $\vec{A}\ X\ (\vec{B}\ X\ \vec{C}) = \vec{B}\ (\vec{A}.\vec{C}) - \vec{C}\ (\vec{A}.\vec{B})$ **3**

3. (a) Attempt any **one:-**

(i) Solve the Legendre's equation $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$ and find $P_n(x)$. **7**

(ii) Derive Bessel's equation from Legendre's Equation. **7**

(b) Attempt any **one:-**

(i) Show $H_n'(x) = 2nH_{n-1}(x)$ **3**

(ii) Find the general solution $\frac{d^2y}{dx^2} + 4y = sin3x$ **3**

4. (a) Attempt any **one:-**

(i) How Fourier transform is represented in three dimensions? Hence Find the Fourier transform of Yukawa potential $f(r) = \frac{e^{-\alpha r}}{r}$ **7**

(ii) Solve the differential equation using Laplace transform **7**

$y'' + 4y = sin\ 2t$ subject to the initial conditions $y_0 = 10,\ y_0' = 0.$

(b) Attempt any **one:-**

(i) State and prove Fourier convolution theorem. **3**

(ii) Find $f(t)$ if Laplace transform $F(s) = \frac{1}{s^2+\frac{s}{2}}$ **3**

5. Attempt any **five:-**

(a) Show that $e^x(x\,cos\,y - y\,sin\,y)$ is a harmonic function. **2**

(b) Find the singularity and its type of $f(z) = sin\,\frac{1}{z-a}$ **2**

(c) Find the values of x, y, z and 'a' which satisfy the matrix equation. **2**

$$\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}$$

(d) Prove that $\epsilon_{ijk}\,\epsilon_{ijk} = 6$ **2**

(e) Show that $P_n(1) = 1$ **2**

(f) Show that $H_n(-x) = (-1)^n H_n(x)$ **2**

(g) Find Laplace transform of $f(t) = t$ using definition. **2**

(h) Find inverse Laplace transform of $\frac{4s-3\pi}{s^2+\pi^2}$ **2**
