Time: 2:30 Hours **Total Marks: 60**

N.B.: (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate **full** marks.
(3) **Symbols** have their usual **meanings** unless otherwise **stated**.
(4) Use of **log tables / non-programmable** calculator is **allowed**.

1. (a) Attempt any **one : -** **8**

(i) The Hamiltonian operator and two other observables $\hat{A}$ and $\hat{B}$ for a certain physical system are represented by matrices

$$\hat{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \hat{A} = \begin{bmatrix} 0 & a & 0 \\ a & 0 & 0 \\ 0 & 0 & 2a \end{bmatrix}, \hat{B} = \begin{bmatrix} 2b & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{bmatrix}$$

Where a and b are real numbers.

A state is given by $|u\rangle = c_1\,|u_1\rangle + c_2|u_2\rangle + c_3\,|u_3\rangle$ where

$$|u_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |u_2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad |u_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

And c_1, c_2, c_3 are constants.

1. Find the relationship between c_1, c_2 *and* c_3 such that $|u\rangle$ is normalized.
2. Find the expectation values of $\hat{H}, \hat{B}$ and $\hat{A}$.
3. What are the possible values of energies that can describe by vector $|u\rangle$?

(ii) Write the answers to the following questions.

1. Define a Hermitian conjugate of a general operator $\hat{A}$ and state the condition for it to be Hermitian.
2. Show that the eigenvalues of Hermitian operators are real.

(b) Attempt any **one : -** **4**

(i) Evaluate

1. $\Delta p_x\,\Delta E$
2. $\Delta x\,\Delta E$

(ii) A linear harmonic oscillator was initially in the state $\psi(x,0) = 2\,\phi_0(x) + i\phi_1(x)$ where ϕ_n are stationary eigenstates with energies $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$

1. Normalize $\psi(x.0)$
2. Find $\psi(x, t > 0)$

2. (a) Attempt any **one : -** **8**

(i) For a linear harmonic oscillator evaluate Δx and Δp_x

(ii) A particle with energy $E > 0$ is incident along x – axis on a potential barrier which is given by

$$V(x) = V_0 \quad -a < x < a$$
$$= 0 \quad otherwise$$

Obtain transmission coefficient for the case $E < V_0$

(b) Attempt any **one : -** **4**

(i) Evaluate $[a, a^\dagger]$, $[a^\dagger, H]$

(ii) A Particle of mass m,which move freely inside an infinite potential well of length a, is initially in the state $\psi(x,0) = \sqrt{\frac{3}{5a}}\sin\left(\frac{3\pi x}{a}\right) + \sqrt{\frac{1}{5a}}\ \sin\left(\frac{5\pi x}{a}\right)$ find

1. $\psi(x,t)$ at any later time, t
2. Calculate the probability density$\rho(x,t)$.

Calculate current density

3. (a) Attempt any **one : -** **8**

(i) The Schrodinger equation for hydrogen atom can be defined as $\frac{d^2u}{d\rho^2} - \frac{l(l+1)}{\rho^2}u + \left(\frac{\lambda}{\rho} - \frac{1}{4}\right)u = 0$. Solve this equation when (a) ρ is very large i.e. $\rho \to \infty$ and (b) ρ is in neighborhood of origin i.e. $\rho \to 0$.

Where $u = rR,\ \rho = 2kr,\ \lambda^2 = \left(\frac{Ze^2}{\hbar} \cdot \sqrt{\frac{\mu}{2|E|}}\right)$.

(ii) Write down Schrodinger equation for two particle system. Redefine the Schrodinger equation in terms of center of mass coordinate $\vec{R}$ and relative coordinate $\vec{r}$. By using separation of variable technique, derive and solve equation for wavefunction corresponding to center of mass $\vec{R}$ of the system.

(b) Attempt any **one : -** **4**

(i) Prove that: $\hat{\vec{L}} \times \hat{\vec{L}} = i\hbar\hat{\vec{L}}$

(ii) Evaluate: $[\hat{L}_x, \hat{x}]$ and $[\hat{L}_x, \hat{y}]$ where $\hat{x}$ and $\hat{y}$ are x-component and y-component of position operator $\hat{\vec{r}}$.

4. (a) Attempt any **one : -** **8**

(i) Derive J_x, J_y and J_z matrices corresponding to angular momentum state $j = 1$.

(ii) For an electron with a spin state $\chi = \frac{1}{\sqrt{13}}\begin{pmatrix}2\\3\end{pmatrix}$, calculate the probability that on measurement the electron will be found in

(1) Spin down state along y-direction

(2) Spin up state along x-direction.

(b) Attempt any **one : -** **4**

(i) Note down all coupled and uncoupled representations for $j_1 = \frac{1}{2}$ and $j_2 = \frac{1}{2}$.

(ii) Evaluate: $[J_z, J_+]$ and $[J_z,\ J_-]$.

5. Attempt any **four: -** **12**

(a) Show that unitary transformations preserve length of vectors

(b) consider the operator A = $\begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

1. show that A is Hermitian
2. Find its eigenvalues.

(c) Show that $\langle x \rangle = 0$ and $\langle p \rangle = 0$ using properties of annihilation and creation operator

(d) Show that Hamiltonian for harmonic oscillator is

$$H = (aa^+ - \frac{1}{2})\hbar\omega$$

(e) Evaluate: $\Delta L_x\, \Delta L_y$

(f) For the wavefunction $\psi = \frac{1}{\sqrt{5}}[2\, Y_3^0 + Y_3^2]$, calculate the expectation value of $\hat{L}_z$ operator.

(g) Prove that all Pauli matrices follow $\hat{\sigma}^2 = \mathbb{I}$ where $\mathbb{I}$ is a 2×2 identity matrix.

(h) Evaluate: $\hat{J}_+ \left| j = \frac{3}{2} \quad m = \frac{1}{2} \right\rangle$.

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