**Time : 2 Hrs** **Marks : 50**

**N.B. :** (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate **full** marks.
(3) Draw **neat** diagrams wherever **necessary**
(4) Symbols have usual meanings unless otherwise stated.
(5) Use of **non-programmable** calculator is allowed.

**1. (a)** Attempt any **one:-**

(i) What is the D Alembert's Principle? Derive Lagrange's equation from D'Alembert's Principle. **7**

(ii) State Hamilton's principle. Derive Lagrangian equation from Hamilton's principle. **7**

**(b)** Attempt any **one:-**

(i) What is velocity dependent potential? Give an example **3**

(ii) By Lagrangian mechanics derive the equation of motion of a single particle in 3 D space using Cartesian coordinates and show that they are equivalent to Newton's laws. **3**

**2. (a)** Attempt any **one:-**

(i) Show that isotropy of space leads to conservation of Angular momentum. **7**

(ii) Obtain the expression for angular momentum and total energy (first integrals) for motion under central force. **7**

**(b)** Attempt any **one:-**

(i) State Kepler's laws of planetary motion **3**

(ii) The maximum and minimum velocities of a satellite are $v_1$ and $v_2$ respectively, find the eccentricity of the orbit of the satellite. **3**

**3. (a)** Attempt any **one:-**

(i) For a particle near the minima of the potential function, show that **7**

$$T_{ij}\ddot{\eta}_j + V_{ij}\eta_j = 0$$

Where symbols have their usual meanings.

(ii) What are Legendre transformations? Derive Hamilton's equations of motion using them. **7**

**(b)** Attempt any **one:-**

(i) What are three types of equilibrium? **3**

(ii) What is cyclic coordinate? Explain with an example. **3**

**4. (a)** Attempt any **one:-**

(i) What are generating functions for canonical transformations. Explain the four types of generating functions. **7**

(ii) What are Poisson's brackets? Show that they remain invariant under canonical transformations. **7**

**(b)** Attempt any **one:-**

(i) Obtain the equation of motion in Poisson bracket form. **3**

(ii) Show that the following transformation is canonical :- **3**

$$P = \frac{1}{2}(p^2 + q^2) \text{ and } Q = \tan^{-1}\left(\frac{p}{q}\right)$$

**5.** Attempt any **five:-**

(a) Obtain the degrees of freedom of a simple pendulum. **2**

(b) Define Holonomic and Non-Holonomic constraints **2**

(c) State Virial theorem. **2**

(d) Explain the terms impact parameter and differential scattering cross section. **2**

(e) Consider the following Lagrangian: **2**

$$L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 - V(r)$$

Which coordinate is cyclic and why?

(f) State and explain variational principle. **2**

(g) Verify whether the transformation $Q = \frac{1}{P}$ and $P = qp^2$ is canonical. **2**

(h) Explain an exact differential condition for a transformation to be canonical. **2**

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