M.Sc. (Part-I) (Mathematics) Second Semester Old

0174 - Mathematical Methods Paper-VI (Optional Paper)

P. Pages : 3 **GUG/W/18/2232**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT - I

1. a) If $f(t) \in C[a, b]$ where $0 < a < b < \infty$ then prove that **10**

$$\int_a^b f(t) \sin(\lambda t) dt \to 0 \text{ as } \lambda \to \infty.$$

b) Find the Fourier sin and cosine transform of $f(t) = e^{-at}$. **10**

OR

c) Determine the temperature u (x, t) in semi infinite rod determine by **10**

$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, 0 \le x \le \infty$ with the conditions.

i) $u = 0$ when $t = 0, x > 0$

ii) $\frac{\partial u}{\partial x} = u$ when $x = 0$

iii) $\frac{\partial u}{\partial x} = 0$ as $x \to \infty$.

d) Prove that $F[(f \cdot g); \xi] = F(\xi) \cdot G(\xi)$ where $F(\xi)$ and $G(\xi)$ denote the Fourier transform of f and g respectively. **10**

UNIT - II

2. a) State and prove existence theorem for Laplace transform. **10**

b) If $L[f(t); P] = \bar{f}(P)$ and $L[g(t); P] = \bar{g}(P)$ then prove that **10**

$$L[f * g; P] = L\left[\int_0^t (t - T) \cdot g(T) dT\right] = \bar{f}(P) \cdot \bar{g}(P).$$

OR

c) If $L[F(t)] = \bar{f}(P)$ then prove that $L[t^n f(t)] = (-1)^n \frac{d^n}{dP^n} \bar{f}(P)$, $n = 1, 2, 3.....$. **10**

d) Solve by Laplace transform. **10**

$y'' - 4y' + 3y = f(x),\ y(0) = 1,\ y'(0) = 0$.

UNIT - III

3. a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 4,\ t > 0$ **10**

subject to the condition u (0, t) = u (4, t) = 0, u (x, 0) = 2x

Give a physical interpretation of the problem.

b) Find the solution of the equation describing the vibrations of a beam of finite length **10**

$\frac{\partial^4 u}{\partial x^4} + \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{p(x,t)}{EI},\ 0 \le x \le a,\ t > 0$. When the displacement u (x, t) satisfies the initial conditions

$u(x,0) = \frac{\partial u}{\partial x}(x,0) = 0,\ 0 \le x \le a$. and the boundary conditions

$u(0,t) = \frac{\partial^2 u}{\partial x^2}(0,t) = 0,\ t > 0$

$u(a,t) = \frac{\partial^2 u}{\partial x^2}(a,t) = 0,\ t > 0$

OR

c) Find the finite Fourier sine transform of cos ax. **10**

d) Express the function. **10**

$$f(x) = \begin{cases} 1 & \text{for} |x| \le 1 \\ 0 & \text{for} |x| > 1 \end{cases}$$

as a Fourier integral and Hence evaluate.

UNIT - IV

4. a) Find the Hankel transform of $X^{\nu-1}$ if $X\, \mathcal{J}_{\nu-1}\ (\xi_n X)$ is the Kernel of transform. **10**

b) Prove that $h_1, \nu\left[f'(x);n\right] = \frac{\nu+1}{2\nu}\, \xi n\, \bar{f}_1,$ **10**

$\nu - 1(n) - \frac{\nu-1}{2\nu}\, \xi n\, \bar{f}_1,\ \nu+1(n), \nu \ne 0$ Where ξ is the root of the equation $\mathcal{J}_n(\xi,\ a) = 0$.

OR

c) Define Mellin transform and calculate. **10**

a) $M\left[e^{-ax}\right], \alpha > 0$ b) $M\left[F(ax)\right]$.

d) Use the Hankel transform of the I[st] kind to find the solution $\theta(r,t)$ of the differential equation. **10**

$$\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}-\frac{1}{k}\frac{\partial\theta}{\partial t}=0$$

where $t \ge 0,\ 0 \le r$ and boundary condition $\theta(a,t)=f(t),\ t \ge 0$ and the initial condition $\theta(r,0)=0$.

5. a) Find the Fourier Sin transform of $f(t)=\frac{e^{-at}}{t}$. **5**

b) Find Laplace $L\left[\frac{\sin\alpha t}{t}\right]$. **5**

c) Find $F_8[X;n]$. **5**

d) Find the Hankel transform of order zero of $\frac{1}{x}$. **5**

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