0133 - Mathematical Physics Paper – I

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1.   EITHER.

a)   Show that $r^n \, \vec{r}$ is an irrotational vector for any of n, but is solenoidal if $n = -3$.   **10**

b)   Define divergence of a vector and give its physical meaning.   **6**

<div align="center">OR</div>

e)   Define curl of a vector. If v is a vector of field then find curl of v in terms of curvilinear Co-ordinates.   **8**

f)   If $\bar{v} = \dfrac{x\vec{i} + y\vec{j} + 2\vec{k}}{\sqrt{x^2 + y^2 + z^2}}$, find the   **8**

value of div $\vec{v}$ and curl of $\vec{v}$

EITHER.

2.   a)   State and prove quotient law of tensor.   **8**

b)   Define Christoffel symbols of first and second kind and prove that.   **8**
$\dfrac{\partial g^{pq}}{\partial x^m} = -g^{pn} \begin{Bmatrix} q \\ mn \end{Bmatrix} - g^{qn} \begin{Bmatrix} p \\ mn \end{Bmatrix}$.

<div align="center">OR</div>

e)   Show that every tensor of second rank can be resolved into symmetric and antisymmetric part.   **10**

f)   Using tensor analysis prove   **6**
   i) $\operatorname{grad}(\phi\psi) = \phi \operatorname{grad}\psi + \psi \operatorname{grad}\phi$.
   ii) $\operatorname{div}(\phi F) = \phi \operatorname{div}F + F\operatorname{grad}\phi$.

3.   EITHER.

a)   State and prove Cayley-Hamilton theorem.   **8**

b)   Find the eigen values and eigen vector of the matrix.   **8**
$A = \begin{bmatrix} b & a & a \\ a & b & a \\ a & a & b \end{bmatrix}$.

e) What are eigen values and eigen vectors? Find the eigen values of the matrix A. **8**

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}.$$

f) Show that all eigen values of a Hermitian matrix are real. **8**

4. EITHER.

a) Find Fourier sine transform of **8**

$$f(x) = \frac{e^{-ax}}{x}.$$

b) Find the inverse Laplace transform of **8**

$$\frac{s^2 - 1}{\left(s^2 + 1\right)^2}$$

OR

e) Prove that for Bessel's function. **8**

$$J_n(x)J_{-n}(x) = (-1)^n J_n(x).$$

f) Find the Fourier series of the function of period 2π defined as **8**

$$f(x) = x \ \text{ if } \ -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$= \pi - x \ \text{ if } \frac{\pi}{2} < x < \frac{3\pi}{2}.$$

5. Answer all the followings.

a) Explain gradient of the scalar function. **4**

b) Define covariant and contravariant tensors of rank 2. **4**

c) Find the inverse of the linear transformation. **4**
   $u = x + 2y - z$
   $v = -2x - y + 3z$
   $w = x - 4z$

d) Prove that **4**

$$\frac{d}{dx}\left[x^n \cdot J_n(x) \right]^2 = x^n J_{n-1}(x).$$

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