M.Sc. – I (Mathematics) First Semester Old

0166 – Optional – Integral Equation Paper – V

P. Pages : 2 **GUG/W/18/2184**

Time : Three Hours Max. Marks : 100

UNIT – I

1. a) Solve the integro differential equation **10**

$$u'(x)+\int_0^1 e^{x-y}u(y)dy = f(x),\ 0\le x\le 1$$

with $u(0)=0$.

b) Find the integral equation formulation for the problem defined by **10**

$\frac{d^2y}{dx^2}+4y=f(x),\ 0\le x\le \frac{\pi}{2}$ with boundary conditions $y=0$ at $x=0$,

$y'=0$ at $x=\frac{\pi}{2}$.

OR

c) Transform the DE $Ly+\lambda r(x)=0,\ x_1\le x\le x_2$ subject to conditions $a_1y(x_1)+b_1y'(x_1)=0$, $a_2y(x_2)+b_2y'(x_2)=0$ into an integral equation. **10**

d) Obtain the integral equation corresponding to **10**

$$y''+2y'+y=0 \text{ with } y(0)=1,\ y'(0)=0.$$

UNIT – II

2. a) Prove that the eigen functions associated with a Hermitian Kernel form an orthonormal set. **10**

b) Find the solution of Fredholm integral equation of second kind whose kernel is Green's function type. **10**

OR

c) Solve the integral equation, **10**

$$3\sin x+2\cos x=\int_{-\pi}^{\pi}\sin(x+y)\phi(y)\,dy$$

d) Find the eigen values & eigen functions of **10**

$$\phi(x)=\lambda\int_0^1(1+xt)\phi(t)\,dt,\ 0\le x\le 1$$

UNIT – III

3. a) Find the Fourier Series for the equation **10**

$$f(x)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{(1-\alpha^2)}{1-2\alpha\cos(x-y)+\alpha^2}\phi(y)\,dy,$$

$0\le\alpha\le 1,\ -\pi<x<\pi$

b) Solve : **10**

$$\int_0^{\pi}\phi(x-y)\left[\phi(y)-2\sin ay\right]dy = x\cos ax.$$

OR

c) Find the resolvent Kernel for the integral equation **10**

$$\phi(x, y) = F(x, y) + \int_0^x \int_0^y e^{(x-\xi)+(y-\eta)} \phi(\xi, \eta) + d\xi d\eta \,,$$

& find the solution.

d) Solve : **10**

$$\phi(x) = 3\int_0^x \cos(x-y)\phi(y)\,dy + e^x \,.$$

UNIT – IV

4. a) Find the two term approximation to the solution of IE. **10**

$$y(x) + \int_0^1 k(x,t)\,y(t)\,dt = 1$$

$$k(x,t) = x,\ x \le t$$

$$= t,\ x \ge t$$

by using Galerkin's method.

b) Solve the IE : **10**

$$\frac{1}{a^2 + x^2} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{g(u)}{x-u}\,du,\ a > 0$$

OR

c) Find the approximation of Kellog's method to the value of smallest eigen value of the Kernel **10**

$$k(x, y) = x,\ \ 0 \le x \le y \le 1$$

$$k(x, y) = y,\ \ 0 \le y \le x \le 1$$

d) Find the first three function in the iterative solution of **10**

$$\phi(x) = \lambda\int_0^1 \sin xy\,\phi(y)\,dy + 1$$

5. a) Show that the IE **5**

$e^{2x} = \int_0^\pi \sin(x+y)\phi(y)\,dy,\ 0 \le x \le \pi$ is not self – consistent & so does not have a solution.

b) Prove that if the eigen values exists they are real. **5**

c) Solve the IE : **5**

$$\int_0^\pi \sin\alpha(x-y)\,\phi(y)\,dy = x$$

d) Solve the IE **5**

$$\int_0^\ell \frac{h(u)}{u-w}\,du = 1\,,\ \ 0 \le w \le \ell$$

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