

CS401 - Applied Mathematics-IV

P. Pages : 2

Time : Three Hours



GUG/W/18/1539

Max. Marks : 80

- Notes : 1. All questions carry equal marks.
2. Non programmable calculator is permitted.

1. a) Prove that 8
i) $A - (B \cup C) = (A - B) \cap (A - C)$ ii) $(A \cup B)' = A' \cap B'$
- b) Let $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$ let R and S be relations from A to B with relation matrices given by - 8
 $M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$, $M_S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$
Find
i) M_R^{-1} and M_S^{-1}
ii) Show that $M_{(R \cap S) \circ R^{-1}} = M_{R \circ R^{-1}} \cap M_{S \circ R^{-1}}$
iii) Find $M_{R^{-1} \cup S^{-1}}$
- OR**
2. a) A TV survey shows that 60% people see programme A, 50% see programme B, 50% see programme C, 30% see programme A and B, 20% see programme B and C, 30% see programme A and C, 10% do not see any programme. 8
i) What percent see programmes A, B, C ?
ii) What percent see exactly two programmes ?
- b) List all possible functions from the set $X = \{a, b, c\}$ to the set $Y = \{0, 1\}$. Indicate in each case whether the function is one-one or onto or both. 8
3. a) Prove by truth table - 8
i) $p \vee (q \vee r) \equiv (p \vee q) \vee r$ ii) $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$
- b) Write converse, inverse, contrapositive and negation of "If two triangles are congruent then their areas are equal." 8
- OR**
4. a) Determine the validity of the following argument. 8
"If Julia does not live in Italy, then She does not speak Italian. Julia does not drive car. If Julia lives in Italy then she travels by train. Either Julia speaks Italian or she drives a car. Therefore, Julia travels by train".
- b) Check for tautology, contradiction and contingency the following : 8
i) $[(p \rightarrow \sim q) \wedge (r \rightarrow q) \wedge r] \rightarrow \sim p$ ii) $(p \vee q) \wedge \sim (p \vee q)$
5. a) Let G be the set of all 2×2 matrices $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, where a and b are real numbers, not both zero such that $a^2 + b^2 \neq 0$ show that G is an abelian group under matrix multiplication. 8

- b) If G is a group then 8
- i) The identity element of G is unique. ii) For every $a \in G$, $(a^{-1})^{-1} = a$
- iii) For every $a, b \in G$, $(a \circ b)^{-1} = b^{-1} \circ a^{-1}$

OR

6. a) Show that $(R_7, +_7, \times_7)$ is a commutative ring with identity. 8
- b) If R is a ring then $\forall a, b \in R$ 8
- i) $a \cdot 0 = 0 \cdot a$ ii) $a \cdot (-b) = (-a) \cdot b = -(a \cdot b)$
- iii) $(-a) \cdot (-b) = a \cdot b$ iv) $a \cdot (b - c) = a \cdot b - a \cdot c$

7. a) Define lattice, Draw the Hasse diagram of lattice D_3 . Write complement of each element of D . 8
- b) Construct switching circuit for the Boolean polynomial $A \cdot B + C + A' \cdot C'$. Simplify this and construct an equivalent circuit verify the equivalence by truth table. 8

OR

8. a) Let S and T be two finite sets such that $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$ then show that $(P(S), \subseteq)$ and $(P(T), \subseteq)$ are isomorphic. 8
- b) Define distributive lattice. Show that every chain (L, \vee, \wedge) is a distributive lattice. 8

9. a) Draw the digraphs corresponding to the adjacency matrix and show that they are isomorphic. 8

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- b) Draw a tree for the relation $R = \{(1, 2), (1, 3), (1, 4), (2, 5), (4, 6), (4, 7)\}$ on a set $A = \{1, 2, 3, 4, 5, 6, 7\}$. Also give the corresponding binary tree. 8

OR

10. a) Construct tree diagram corresponding to algebraic expression 8
- i) $(x + (y + z)) - (a \times (b + c))$
- ii) $(3 - 2(-11 - (9 - 4))) \div (2 + (3 + (4 + 7)))$

- b) Using Prim's algorithm find the minimal spanning tree of - 8


