



- Notes : 1. All questions carry equal marks.
2. Use of non programmable calculator is permitted.

1. a) i) Find $L\{(t^2 - 1)\sin t\}$. 4

ii) Find Laplace transform of $\frac{\cos at - \cos bt}{t}$ hence evaluate $\int_0^\infty \left\{ \frac{\cos at - \cos bt}{t} \right\} dt$ 5

b) Express $f(t) = \begin{cases} \cos t & , 0 < t < \pi \\ \cos 2t & , \pi < t < 2\pi \\ \cos 3t & , t > 2\pi \end{cases}$ 7
in terms of unit step function and hence find its Laplace transform.

OR

2. a) Use convolution theorem to find $L^{-1}\left\{ \frac{s^2}{(s^2 + a^2)(s^2 + b^2)} \right\}$ 8

b) Find $y(t)$ using Laplace Transform if $\frac{dy}{dt} + 2y + \int_0^t y dt = \sin t$ given $y(0) = 1$. 8

3. a) Find A^{-1} by Partitioning if $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ 8

b) Find the value of λ for which the following equations are consistent. Also solve the system for these value of λ 8
 $x + 2y + z = 3$
 $x + y + z = \lambda$
 $3x + y + 3z = \lambda^2$

OR

4. a) Find the matrix which diagonalize the matrix $A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$ hence find diagonal matrix. 8

b) Find the rank of matrix $\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$ 4

c) Investigate whether the vectors $x_1 = (1, 2, 4)$ $x_2 = (2, -1, 3)$ $x_3 = (0, 1, 2)$, $x_4 = (-3, 7, 2)$ are linearly dependent or not. 4
Also find relation between them.

5. a) If $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$ then show that $\operatorname{cosec}^2 A - \cot^2 A = I$ by Sylvester's theorem. 8
- b) Verify Cayley-Hamilton theorem for given matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ hence find 8
- $$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$$

OR

6. a) Solve $\frac{d^2 y}{dt^2} + 4y = 0$ given $y = 8, \frac{dy}{dt} = 0$ when $t = 0$ by matrix method. 8
- b) Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 + 4x_1x_3 - 8x_2x_3$ to canonical form by an orthogonal transformation. 8
7. a) Solve $z(p-q) = z^2 + (x+y)^2$ 4
- b) Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ 4
- c) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \tan(x+y) + e^{x+y}$ 8

OR

8. a) Solve $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 2\cos y - x\sin y$ 8
- b) Solve using method of separation of variable $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = \mu$, given that 8
- $$\mu(x, 0) = 3e^{-5x} + 2e^{-3x}$$
9. a) Find the Fourier series for $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & , -\pi < x < 0 \\ 1 - \frac{2x}{\pi} & , 0 < x < \pi \end{cases}$ 8
- hence show that $f(x) = \frac{8}{\pi^2} \left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots \right]$
- b) Find half range cosine series for $f(x) = (x-1)^2, 0 < x < 1$. 8

OR

10. a) Using Fourier integrals show that $\int_0^\infty \frac{(1 - \cos \pi \lambda) \sin \lambda x}{\lambda} d\lambda = \begin{cases} \pi/2 & 0 < x < \pi \\ 0 & x > \pi \end{cases}$ 8
- b) Solve the integral equation $\int_0^\infty f(x) \cos \alpha x dx = \begin{cases} 1 - \alpha & 0 \leq \alpha \leq 1 \\ 0 & \alpha > 1 \end{cases}$ 8
