B.E. Electrical (Electronics & Power) Engineering / Electronics Engineering / Electronics & Telecomm. / Comm. Engineering / Instrumentation Engineering
Third Semester (Old Pattern)

EP/EN/ET/IN-301 : Applied Mathematics - III

P. Pages : 2 **GUG/W/18/1483**

Time : Three Hours Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of non programmable calculator is permitted.

1. a) Find $L\left\{\frac{e^{-3t}-e^{-6t}}{t}\right\}$ Hence evaluate $\int_0^{\infty}\frac{e^{-3t}-e^{-6t}}{t}\,dt$ **8**

b) Express $f(t)=\begin{cases} t^2 & , \quad 0<t<1 \\ 4t & , \quad t>1 \end{cases}$ **8**

in terms of unit step function and hence obtain its Laplace transform.

OR

2. a) Use convolution theorem to find $L^{-1}\left\{\frac{1}{(s+1)\,(s^2+1)}\right\}$ **8**

b) Solve, by Laplace transform method $\frac{d^2y}{dt^2}+9y=\sin t$ **8**

given that $y(0)=1,\ y\left(\frac{\pi}{2}\right)=-1$.

3. a) Find the inverse of a matrix $A=\begin{bmatrix}1 & 3 & 3\\ 1 & 3 & 4\\ 1 & 4 & 3\end{bmatrix}$ by the method of partitioning. **8**

b) Test for consistency and solve **8**

$x+y+z=6$

$x-y+2z=5$

$3x+y+z=8$

OR

4. a) Find the modal matrix B corresponding to the matrix $A=\begin{bmatrix}3 & -1 & 1\\ -1 & 5 & -1\\ 1 & -1 & 3\end{bmatrix}$ **8**

Also write down the diagonal matrix.

b) If $\xi = x\cos\alpha - y\sin\alpha$ **5**

and $\eta = x\sin\alpha + y\cos\alpha$

prove that the transformation is orthogonal and hence write the inverse transformation.

c) Investigate linear dependence or independence of the vectors **3**

$x_1=(1,\ 2,\ 2)$

$x_2=(2,\ 1,\ -2)$

$x_3=(2,\ -2,\ 1)$

5. a) Verify Cayley Hamilton's theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ **8**

and hence obtain A^{-1}.

b) Use Sylvester's theorem to show that sin A = (sin 1) **8**

A where $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

OR

6. a) Solve, by matrix method $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$ given that x(0) = 2 and $\frac{dx}{dt} = 0$ at t = 0. **8**

b) Find the largest eigen value and the corresponding eigen vector of the following matrix, **8**

by iterative method $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

7. Solve

a) $x\frac{\partial z}{\partial y} = y\frac{\partial z}{\partial x} + x\,e^{x^2+y^2}$ **3**

b) $(2z - y)\frac{\partial z}{\partial x} + (x + z)\frac{\partial z}{\partial y} + 2x + y = 0$ **5**

c) $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x\,\partial y} - 3\frac{\partial^2 z}{\partial y^2} = \sin(2x + 3y) + xy$ **8**

OR

8. a) Solve $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x\,\partial y} - 8\frac{\partial^2 z}{\partial y^2} = e^{2x+y} + \sqrt{2x + 3y}$ **8**

b) Solve, by the method of separation of variables $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = u$ **8**

given that $u(x, 0) = 3e^{-5x} + 2e^{-3x}$

9. a) Find the Fourier series for $f(x) = 2x - x^2,\ 0 < x < 2$. **8**

b) Find the half range cosine sines for $f(x) = \sin x,\ 0 \le x \le \pi$. **8**

OR

10. a) Express $f(x) = \begin{cases} 1, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}$ as a Fourier integral and hence evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$ **8**

b) Find the Fourier sine transform of $e^{-|x|}$ and hence show that **8**

$\int_0^\infty \frac{x \sin mx}{1 + x^2}\,dx = \frac{\pi}{2}e^{-m},\ m > 0$

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