B.E. Computer Science & Engineering / Computer Technology / Information Technology
Third Semester

# CSE/CT/IT-301 : Applied Mathematics-III

P. Pages : 3 **GUG/W/18/1473**

Time : Three Hours * 1 1 6 2 * Max. Marks : 80

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Notes : 1. All questions carry equal marks.
2. Use of non programmable calculator is permitted.

**1.** a) If $z\{f(n)\} = F(z)$ then prove that $z\{n\,F(n)\} = -z\frac{d}{dz}\{F(z)\}$ hence find $z\{n\,_z{}^n\}$. **8**

b) Solve by z - transform $y_{n+2} + 4y_{n+1} + 4y_n = 3^n$ given $y_0 = y_1 = 0$. **8**

**OR**

**2.** a) Express $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ as Fourier integral and hence evaluate $\int_0^\infty \frac{\sin\lambda}{\lambda} d\lambda$ **8**

b) Solve the integral equation $\int_0^\infty f(x)\cos\alpha x\,dx = \begin{cases} 1-\alpha & 0 \le \alpha \le 1 \\ 0 & \alpha > 1 \end{cases}$ **8**

**3.** a) Find the inverse of matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ by method of Partitioning. **8**

b) Show that the equations **8**

$-2x + y + z = a$

$x - 2y + z = b$

$x + y - 2z = c$

do not have solution unless a + b + c = 0.
Hence find the solution of a = 1, b = 1, c = 2

**OR**

**4.** a) Test for linear dependence the vectors $X_1 = (1, 1, 1, 3)$, $X_2 = (1, 2, 3, 4)$, $X_3 = (2, 3, 4, 9)$. **5**

b) Find the rank of matrix $\begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$ **3**

c) Find the modal matrix B corresponding to matrix $A = \begin{bmatrix} 0 & 1 \\ 12 & -4 \end{bmatrix}$ and verify that $B^{-1}AB$ is diagonal matrix. **8**

**5.** a) Verify Cayley - Hamilton theorem for matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find $A^{-1}$. **8**

b) Show that $3\tan A = (\tan 3)A$ **8**

where $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$

**OR**

**6.** a) Find the largest eigen value and the corresponding eigen vector for the matrix **8**

$\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$

b) Solve by matrix method $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} - 6y = 0$ given that y = 2, $\frac{dy}{dt} = 5$ at t = 0. **8**

**7.** a) An urn holds 5 white and 3 black marbles. If two marbles are drawn at random without replacement and X denotes the number of white marbles : **8**

Find :

i) Probability function

ii) Distribution function

b) If the joint density function of random variable X and Y is given by **8**

$$f(x, y) = \begin{cases} c(2x + y) & 2 < x < 6, \quad 0 < y < 5 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find :

i) The constant C

ii) Marginal distribution function of X and Y

iii) Marginal density function of X and Y.

**OR**

**8.** a) The distribution function of a random variable X is given by **8**

$$F(x) = \begin{cases} cx^3 & , \quad 0 \le x < 3 \\ 1 & , \quad x \ge 3 \\ 0 & , \quad x < 0 \end{cases}$$

Find :

i) C ii) density function iii) P(X > 1)

b) The joint probability function of two discrete random variables X and Y is given by **8**

$$f(x, y) = \begin{cases} cxy, & x = 1, 2, 3 \quad y = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

Find :

i) Constant C ii) $P(x \ge 2)$

iii) The marginal probability function of both X and Y

iv) Check whether X and Y are independent.

**9.** a) Find moment generating function and first four moments about origin for random variable X given by **8**

$$X = \begin{cases} 1 & ; \text{ Prob } ½ \\ -1 & ; \text{ Prob } ½ \end{cases}$$

b) The random variable X and Y have joint density function given by **8**

$$f(x, y) = \begin{cases} \dfrac{(2x+y)}{210} & ; \quad 2 < x < 6 \,,\, 0 < y < 5 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Find :

i) Var (X) ii) Var (Y)

iii) Cov(X, Y) iv) Coefficient of correlation ρ.

**OR**

**10.** a) The joint probability function for random variable X and Y is given in the table. **8**

| X / Y | 0 | 1 | 2 | |
|---|---|---|---|---|
| 0 | 1/18 | 1/9 | 1/6 | 6/18 |
| 1 | 1/9 | 1/18 | 1/9 | 5/18 |
| 2 | 1/6 | 1/6 | 1/18 | 7/18 |
| | 6/18 | 6/18 | 6/18 | G.T. = 1 |

Find :

i) Var(X) ii) Var(Y)

iii) $\sigma_{XY}$ iv) Coefficient of correlation ρ .

b) Find the coefficients of i) Skewness ii) Kurtosis of the distribution - **8**

$$f(x) = \begin{cases} e^{-x} & ; \quad x \geq 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

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