B.E. Second Semester (Old) (C.B.S. Pattern)

101 - Applied Mathematics-II

P. Pages : 3 | GUG/W/18/1454

Time : Three Hours | Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of Non-programmable calculator is permitted.

1. a) Solve $1+\log(xy)+\left(1+\frac{x}{y}\right)\frac{dy}{dx}=0$ **5**

b) Solve $ye^{y}dx=\left(y^{3}+2xe^{y}\right)dy.$ **5**

c) Solve $\frac{d^{2}y}{dx^{2}}+6\frac{dy}{dx}+9y=\frac{e^{-3x}}{x^{2}}.$ **6**

OR

2. a) Solve $x\left(\frac{dy}{dx}+y\right)=1-y.$ **4**

b) Solve $y\left(1+\frac{1}{x}\right)+\cos y+(x+\log x-x\sin y)\frac{dy}{dx}=0.$ **5**

c) Solve by variation of parameter. **7**

$\frac{d^{2}y}{dx^{2}}+y=\operatorname{cosec} x.$

3. a) Solve $\frac{d^{2}y}{dx^{2}}+2y=x^{2}e^{3x}+e^{x}\cos 2x.$ **8**

b) Solve $x^{2}\frac{d^{2}y}{dx^{2}}+x\frac{dy}{dx}+4y=\log x\cos(\log x)+\frac{1}{x}.$ **8**

OR

4. a) Solve $\frac{dx}{dt}+3x-2y=1.$ **8**

and $\frac{dy}{dt}-2x+3y=e^{t}.$

given that $x=0, y=0$ at t = 0.

b) Solve $\frac{d^2y}{dx^2} = 2\left(y^3 + y\right)$ **8**

given that $y = 0, \frac{dy}{dx} = 1$ when x = 0.

5. a) Evaluate, by changing the order of integration. **8**

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx.$$

b) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy.$ **8**

OR

6. a) Find the Centre of gravity of the area between $y = 6x - x^2$ and $y = x$. **8**

b) Find the mass of tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, the variable density $\rho = k\,xyz$. **8**

7. a) Show that the tangent vector to the curve **4**

$x = t^2 + 2, \; y = 2t^2 - 1, \; z = 2t^2 - 6t$ at $t = \pm 1$ are orthogonal.

b) A particle moves along the curve **6**

$\vec{r} = \cos wt\,\hat{i} + \sin wt\,\hat{j}.$

where w is constant and t is the time.

Show that

i) $\vec{r}$ is perpendicular to $\vec{v}$

and ii) $\vec{r} \times \vec{v} =$ constant vector.

c) Find the directional derivative of **6**

$\phi = 4e^{2x-y+z}$ at the point $(1,1,-2)$ in the direction towards the point $(-3,5,6)$.

OR

8. a) If $\vec{r} = \vec{a}\cos wt + \vec{b}\sin wt$. **4**

Then show that

i) $\vec{r} \times \frac{d\vec{r}}{dt} = w\left(\vec{a} \times \vec{b}\right)$

and ii) $\frac{d^2\vec{r}}{dt^2} = -w^2\,\vec{r}$.

b) A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5,$ where t in time. Find the components of its velocity and acceleration at t = 1 in direction $\overrightarrow{AB}$ if $A(1,2,1)$ & $B(2,-1,3)$. **6**

c) Find the values of a and b so that the surfaces $ax^2 - byz = (a+2)^x$ and $4x^2y + z^3 = 4$ are orthogonal at the point $(1,-1,2)$. **6**

9. a) Find the value of n for which the vector field $r^n \vec{r}$ will be solenoidal. Find also whether the vector field $r^n \vec{r}$ is irrotational or not. **8**

b) Show that the vector field **8**

$$\vec{F} = \left(y^2 \cos x + z^3\right)\hat{i} + (2y \sin x - 4)\hat{j} + \left(3xz^2 + 2\right)\hat{k}$$

is irrotational. Also find its scaler potential ϕ.

OR

10. a) Evaluate, by Gauss-Divergence Theorem **8**

$$\iint_S \vec{F} \circ \hat{n}\, ds.$$

where $\vec{F} = 4x\,\hat{i} - 2y^2\,\hat{j} + z^2\,\hat{k}$

and S is closed surface bounded by.

cylinder $x^2 + y^2 = 4$ and planes z = 0 and z = 3.

b) Evaluate, by stokes theorem **8**

$$\int_C \vec{F} \circ d\,\vec{r}.$$

where $\vec{F} = (2x + y)\,\hat{i} - 4z^2\,\hat{j} - y^2\,\hat{k}.$

and C is boundary of circle $x^2 + y^2 + z^2 = 1,\ z = 0.$

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