B.E. I & II First / Second Semester Old (CBS Pattern)

105 - Applied Mathematics-I

P. Pages : 2 **GUG/W/18/1450**

Time : Three Hours * 1 1 4 8 * Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) If $Y = \tan^{-1}\left(\frac{x}{a}\right)$ then show that $Y_n = \frac{(-1)^{n-1}(n-1)!}{a^n}\sin^n\theta \sin n\theta$ where $\theta = \tan^{-1}\left(\frac{a}{x}\right)$ **8**

b) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{b}\right)^n$, Prove that $x^2 y_{n+2} + (2n+1)x\, y_{n+1} + 2n^2 y_n = 0$ **8**

OR

2. a) If $Y = \left[x + \sqrt{1+x^2}\,\right]^m$, Prove that $(1+x)^2 y_{n+2} + (2n+1)x\, y_{n+1} + (n^2 - m^2) y_n = 0$
Hence find $(y_n)_0$ **8**

b) Using Taylor's theorem, find the value of $\cos 64^o$ correct to four decimal places. **4**

c) Evaluate $\lim_{x\to 1} \frac{x^x - x}{1 - x + \log x}$ **4**

3. a) If $u = \log(x^3 + y^3 - x^2 y - xy^2)$, show that $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial^2 u}{\partial y^2} = \frac{-4}{(x+y)^2}$ **8**

b) If $u = x\phi\left(\frac{y}{x}\right) + \psi\left(\frac{y}{x}\right)$, Prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$ **8**

OR

4. a) If $u = \frac{(x^2+y^2)^m}{2m(2m-1)}$ then Prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \left(x^2+y^2\right)^m$. **8**

b) If $u = F(x, y)$, $X = e^r\cos\theta$, $y = e^r\sin\theta$ then prove that **8**
$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2r}\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial \theta}\right)^2\right]$

5. a) If $u = x^m y^n$ $v = x^r y^s$ then show that $\frac{\partial(x,y)}{\partial(u,v)} = \frac{xy}{(ms - rn)uv}$ **8**

b) Find the volume of the greatest rectangular Parallelepiped that can be inscribe in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ **8**

OR

6. a) If $u=\frac{x+y}{1-xy}$ and $v=\tan^{-1}x+\tan^{-1}y$, Are u and v functionally related? If so find relation between them. **8**

b) Expand f(x,y) = sin(xy) in power of (x – 1) and $\left(y-\pi/2\right)$ upto second degree terms **8**

7. a) Given $\sqrt{1.8} = 0.9314$ Find the value of $\sqrt{-2.2}$. **4**

b) Prove that $\int_0^{\pi/2}\sqrt{\tan\theta}\ d\theta=\frac{\pi}{\sqrt{2}}$ **4**

c) Evaluate $\int_0^1\frac{x^\alpha-1}{\log x}\ dx,\ \alpha\geq 0$ by applying Differentation under the integral sign. **8**

OR

8. a) Find the roat mean square value of $f(t)=3\sin 2t+4\cos 2t$ over the range $0\leq t\leq\pi$. **8**

b) Trace the curve $3ay^2=x(x-a)^2$ Also find **8**

i) The perimeter of the loop of the curve.
ii) The area of the loop of the curve

9. a) Two lines of regression are given by x + 2y – 5 = 0 and 2x + 3y – 8 = 0 if $\sigma_x^2=12$ Find **8**

i) The mean value of x and y
ii) The standard deviation of y
iii) The coefficient of correlation between x and y.

b) Fit the curve $y=ax^b$ to the following data by least square method **8**

x :	1	2	3	4	5	6
y :	2.98	4.26	5.21	6.10	6.80	7.50

OR

10. a) Fit a second degree parabola $y:a+bx+cx^2$ by least square method to the following data. **8**

x :	20	30	40	50	60
y :	54	90	138	206	292

b) Calculate the rank correlation coefficient for the following data. **8**

x:	60	34	40	50	45	40	22	43	42	66	64	46
y:	75	32	33	40	45	33	12	30	34	72	41	57

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