Bachelor of Science (B.Sc.-III) Sixth Semester

B.Sc. 4532 - Mathematics Paper-II (Optional)
(Number Theory and Discrete Mathematics)

P. Pages : 3 **GUG/W/18/1355**

Time : Three Hours * 1 0 6 8 * Max. Marks : 60

Notes : 1. Solve **all five** questions.
2. Q. 1 to Q. 4 have an alternative solve each question in full or its alternative in full.
3. All questions carry equal marks.

UNIT – I

1. a) Find a solution of **6**
$8x + 7y = 89$

b) Find a general solution of **6**
$12x + 7y = 122$

OR

c) Prove that **6**
$ax + by = a + c$ is solvable if and only if $ax + by = c$ is solvable.

d) State and prove Wilson's theorem. **6**

UNIT – II

2. a) Find all integer that satisfies simultaneously **6**

$$x \equiv 2 \pmod 3$$
$$x \equiv 3 \pmod 5$$
$$x \equiv 5 \pmod 2.$$

b) Find all integer that give remainders 1, 2, 3 when divided by 3, 4, 5 respectively. **6**

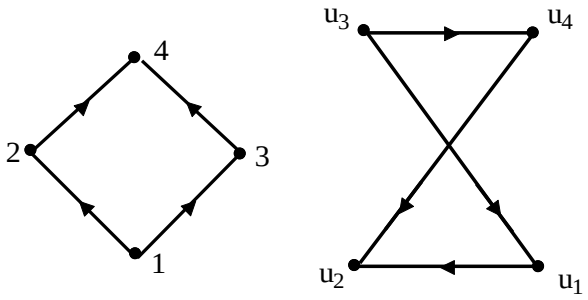
OR

c) Solve $x^2 + x + 7 \equiv 0 \pmod{189}$. **6**

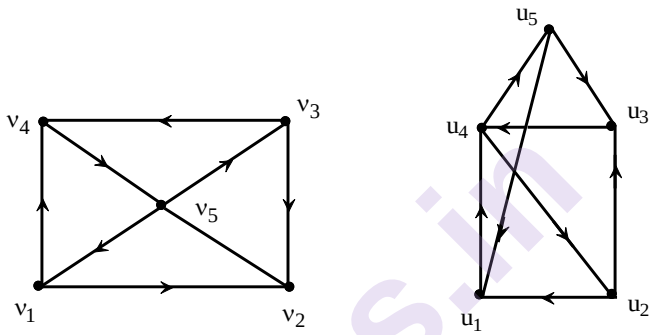
d) Prove that **6**
$ax \equiv b \pmod m,\ (a, m) = 1$
has only one solution.

UNIT – III

3. a) Show that the following graphs are isomorphic. **6**

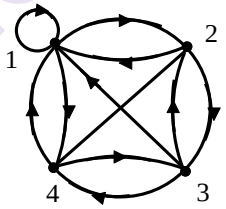


b) Show that the digraphs given below are isomorphic. **6**

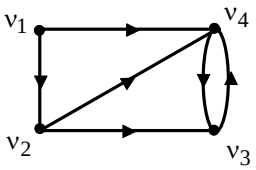


OR

c) Find the path originating in node 1 and ending in node 3 and some cycles of given diagraph. **6**



d) Find a node-base for the digraph given below. **6**



UNIT – IV

4. a) Show that in a lattice if $a \le b$ and $c \le d$, then $a * c \le b * d$. **6**

b) Show that in a lattice, **6**
$a \le b \Leftrightarrow a \oplus b = b$.

OR

c) Prove that the Boolean identities **6**

i) $a \oplus (a' * b) = a \oplus b$.

ii) $a * (a' \oplus b) = a * b$.

iii) $(a * b * c) \oplus (a * b) = a * b$.

d) Simplify the Boolean expression. **6**

i) $(a*c)\oplus c\oplus\left[(b\oplus b')*c\right]$

ii) $(a'*b'*c)\oplus(a*b'*c)\oplus(a*b'*c')$.

5. Solve **any six.**

a) Let a, b, c, d, x, y denote integers then show that **2**
$a\equiv b(\mathrm{mod}\,m), b\equiv a(\mathrm{mod}\,m)$ and
$a-b\equiv 0(\mathrm{mod}\,m)$ are equivalent.

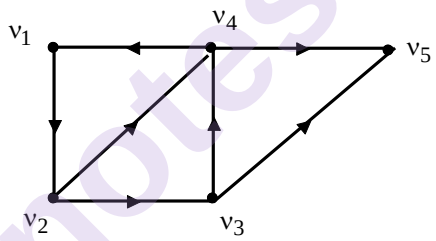
b) Exhibit a reduced residue system for the modulo 12 and 30. **2**

c) Solve congruence. **2**
$x^2+2x-3\equiv 0(\mathrm{mod}\,9)$

d) Solve $x^3+4x+8\equiv 0(\mathrm{mod}15)$ **2**

e) Define Reachable set. **2**

f) Find all the indegrees and outdegrees of the digraph given below **2**



g) Let the sets $S_0, S_1, ---, S_7$ be **2**
Given by

$S_0=\{a,b,c,d,e,f\};$

$S_1=\{a,b,c,d,e\};$

$S_2=\{a,b,c,e,f\};$

$S_3=\{a,b,c,e\};$

$S_4=\{a,b,c\}$

$S_5=\{a,b\};$

$S_6=\{a,c\};$

$S_7=\{a\}$

Draw diagram of $(L,\subseteq)$

Where $L=\{S_0,S_1,----S_7\}$.

h) Define Hasse diagram. **2**

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