Bachelor of Science (B.Sc.)-III Sixth Semester

B.Sc. 4531 / MAT 305 - Mathematics-II
(Optional) (Special Relativity-II) Paper - II

P. Pages : 2 **GUG/W/18/1354**

Time : Three Hours * 1 0 6 7 * Max. Marks : 60

Notes : 1. Solve all the **five** questions.
2. Question 1 to 4 has an alternative. Solve each question in full or its alternative in full.
3. All questions carry equal marks.

UNIT – I

1. a) Define contravariant tensor of order one. Show that gradient of a scaler is a covariant vector. **6**

b) A covariant vector has components $2x-z,\ x^2y,\ yz$ in rectangular co-ordinates. Find its covariant components in cylindrical co-ordinates. **6**

OR

c) Show that, if A_{rs} and B^{pq}_{m} are tensors, $A_{rs}\,B^{pq}_{m}$ is a tensor. **6**

d) Let A^r be an arbitrary tensor defined by its indices. If the inner product $A^r B_r$ is an in variant, then prove that B_r is a covariant tensor of order one. **6**

UNIT – II

2. a) Prove that $g_{,r}^{mn} = -g^{ms}\left\lceil\!\!\left\rceil^{n}_{sr} - g^{sn}\right\lceil\!\!\left\rceil^{m}_{sr}\right.\right.$ **6**

b) Prove that the covariant derivatives of $g_{mn,}\ g^{mn}$ and δ^m_n vanish. **6**

OR

c) Define geodesics. Show that the geodesics in three dimensional Euclidean space are straight lines. **6**

d) If $R^{P}_{,\,rmn}$ and R_{prmn} are curvature tensors there prove that **6**

i) $R^{P}_{,\,rmn} = -R^{P}_{,\,rnm}$

ii) $R^{P}_{,\,rmn} + R^{P}_{,\,mnr} + R^{P}_{,\,nrm} = 0$

iii) $R_{prmn} = -R_{rpmn}$

UNIT – III

3. a) Obtain the mass energy equivalence relation $E = mc^2$. **6**

b) A particle is given a kinetic energy equal to n times its rest energy $m_o c^2$. What are its speed and momentum. **6**

OR

c) A body of mass m disintegrates into parts m_1 and m_2, while at rest. Show that the energies E_1 and E_2 of the two parts satisfy the relation **6**

$$\frac{E_1}{E_2} = \frac{m^2 + m_1^2 - m_2^2}{m^2 - m_1^2 + m_2^2}$$

d) Prove that the four force can be expressed as **6**

$$f^i = \left(\frac{\bar{F}}{c\sqrt{1 - u^2/c^2}}, \frac{\bar{f} \cdot \bar{u}}{c^2\sqrt{1 - u^2/c^2}} \right)$$

UNIT – IV

4. a) Explain the term 'four potential'. Obtain transformation of the electromagnetic four potential. **6**

b) Show that the Hamiltonian for a charged particle moving in an electromagnetic field is **6**

$$H = \left\{ m_0^2 c^4 + c^2 \left(P - \frac{e}{c} A \right)^2 \right\}^{1/2} + e\,\phi$$

OR

c) Obtain transformation for electric field strength F_{14}^1. **6**

d) Prove that energy momentum tensor of electromagnetic field is trace tree. **6**

5. Solve **any six.**

a) Define Mixed tensor of order two. **2**

b) Prove that $\delta_s^r A^s = A^r$. **2**

c) Define Christoffel symbols of the first kind. **2**

d) Define Ricci tensor and Einstein tensor. **2**

e) Define four velocity and four acceleration. **2**

f) Prove that four velocity of a particle is a unit time like vector. **2**

g) Define electromagnetic field tensor. **2**

h) State the Maxwell's equations of electromagnetic theory in vacuum. **2**

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