Bachelor of Science (B.Sc.-III) Sixth Semester

B.Sc. 4530 – Mathematics Compulsory Paper – I (Analysis)

P. Pages : 2 **GUG/W/18/1353**

Time : Three Hours Max. Marks : 60

Notes :
1. Solve all the **five** questions.
2. Question 1 to 4 has an alternative solve each question in full or its alternative in full.
3. All questions carry equal marks.

UNIT – I

1. a) Let X be an arbitrary non – empty set. Defined by **6**

$$d(x,y)=\begin{cases}0 \text{ for } x=y\\1 \text{ for } x\neq y\end{cases}$$

Show that d is a metric on X.

b) If p is a limit point of a set A. Then prove that every neighbourhood of p contains infinitely many points of A. **6**

OR

c) Let Y be a subspace of a complete metric space X. Then prove that **6**

Y is complete $\Leftrightarrow Y$ is closed.

d) Prove that closed subsets of compact sets are compact. **6**

UNIT – II

2. a) Let f, g be bounded functions defined on [a, b] and P be any partition of [a, b]. Then prove that **6**

$U(p,f+g)\leq U(p,f)+U(p,g)$

and $L(p,f+g)\geq L(p,f)+L(p,g)$

b) Let the function f be defined as **6**

$$f(x)=\begin{cases}1, & x \text{ is rational}\\-1, & x \text{ is irrational}\end{cases}$$

Show that f is not R – integrable over [0, 1] but $|f|\in R[0,1]$.

OR

c) Prove that every continuous function is integrable. **6**

d) If f is continuous on [a, b] and F is continuous and differentiable on [a, b] with $F'(x)=f(x),\ x\in[a,b]$ then prove that **6**

$$\int_a^b f(x)\,dx=F(b)-F(a)$$

UNIT – III

3. a) Evaluate $\int_{1-i}^{2+i}(2x+iy+1)\,dz$ along the straight line joining the points $(1-i)$ and $(2+i)$. **6**

b) Let C_1 be a simple closed contour and let C_2 be another simple closed contour lying entirely inside C_1. If $f(z)$ is analytic in the ring shaped domain between C_1 and C_2. Then prove that $\int_{C_1} f(z)\,dz = \int_{C_2} f(z)\,dz$. Where C_1 and C_2 have the same orientation direction. **6**

OR

c) Evaluate $\int_C \frac{15z+9}{z(z^2-9)}\,dz$, where C is the circle $|z-1|=3$. **6**

d) Show that the function cosec (1/z) has a non – isolated essential singularity at Z = 0. **6**

UNIT – IV

4. a) Find the Finite Fourier sine and finite Fourier cosine transform of the function $f(x)=\pi$ in the interval $0 < x < 2$. **6**

b) Find the finite Fourier sine and cosine transform of $f(x)=\sin ax$ in the interval $(0, \pi)$. **6**

OR

c) Show that the Fourier transform of $f(ax+b)$ is $\frac{1}{a} e^{-i\lambda b/a} F[\lambda / a]$. **6**

d) Find the Fourier sine transform of $f(x)=\frac{e^{-ax}}{x}$, $a>0$. Hence evaluate $\int_0^{\infty} \tan^{-1}\frac{x}{a} \sin x\,dx$. **6**

5. Attempt **any six.**

a) Define a metric on a nonempty set X. **2**

b) Define Cauchy Sequence. **2**

c) Prove that **2**
$m(b-a) \le L(p,f) \le U(p,f) \le M(b-a)$
where M and m denote the sup and inf of $f(x)$ in $I=[a,b]$.

d) Give an example of a Riemann integrable function on [a, b] which is not monotonic on [a, b]. **2**

e) Show that $\int_C \left(\frac{1}{z}\right) dz = 2\pi i$ where C is the circle with centre at the origin and radius r. **2**

f) Define zero of an analytic function and singular point of $f(z)$. **2**

g) Find finite Fourier sine transform of the function $f(x)=K,\ K \in N$ where $0 < x < \ell$. **2**

h) If $F[f(x)]=F(\lambda)$, then prove that $F[f(x-a)]=e^{-i\lambda a}\,F(\lambda)$. **2**

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