Bachelor of Science (B.Sc.)-III Fifth Semester

B.Sc. 3528 / MAT 301 - Mathematics Paper-I (Compulsory) (Linear Algebra)

P. Pages : 2 **GUG/W/18/1315**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that a necessary condition that $F(z) = u(x, y) + iv(x, y)$ be analytic in a region D is that $u_x = v_y$ and $u_y = -v_x$ in D. **6**

b) If $f(z)$ and $f(\bar{z})$ are analytic functions prove that f(z) is constant. **6**

OR

c) Find the fixed points of the bilinear transformation $W = \frac{(2+i)z-2}{i+z}$. What is its normal form? Show that the transformation is loxodromic. **6**

d) Prove that the cross ratio remains invariant under a bilinear transformation. **6**

UNIT – II

2. a) Prove that a non-empty subset U of a vector space V over F is a subspace of V if and only if **6**

i) $u + v \in U \ \forall u, v \in U$ and ii) $\alpha u \in U \ \ \forall \alpha \in F, \ u \in U$

b) Let U, W be subspaces of a vector space V(F). Prove that $U \cup W$ is a subspace of V if and only if $U \subseteq W$ or $W \subseteq U$. **6**

OR

c) Let $v_1, v_2, ..., v_n$ be n vectors of a vector space V(F). Prove that **6**

i) $[v_1, v_2, ...v_n] = [\alpha_1 v_1, \alpha_2 v_2 ....\alpha_n v_n], \ \alpha_i (\neq 0) \in F \forall i$

ii) $[v_1, v_2] = [v_1 - v_2, \ v_1 + v_2]$

d) Prove that the set $B_1 = \{ (1, 1, 1), (1, -1, 1), (0, 1, 1)\}$ is a basis of v_3. **6**

UNIT – III

3. a) Let U, V be vector spaces over the same field F. then prove that a function $T : U \to V$ is linear if and only if $T(\alpha u + \beta v) = \alpha T(u) + \beta T(v), \ \forall u, v \in U$ and $\alpha, \beta \in F$. **6**

b) If $T : V_2 \to V_4$ be a linear map defined by $T(1,1) = (0, 1, 0, 0), \ T(1, -1) = (1, 0, 0, 0)$ where $\{ (1, 1), (1, -1)\}$ is basis of V_2. Find T(x , y). **6**

OR

c) Let $T: U \to V$ be linear map. then prove that **6**
i) $R(T)$ is a subspace of V. ii) $N(T)$ is a subspace of U.

d) Prove that a linear map $T: V_3 \to V_3$ defined by **6**
$T(e_1) = e_1 - e_2$, $T(e_2) = 2e_2 + e_3$
$T(e_3) = e_1 + e_2 + e_3$ is neither $1-1$ nor onto.

UNIT – IV

4. a) Let V be an inner product space over F. If $U, v \in V$ then prove that $|(U, V)| \le \| U \| \| V \|$. **6**

b) Let V be a set of all continuous complex-valued function on the closed interval [0,1]. If $F(t), g(t) \in V$ define. $(F(t), g(t)) = \int_0^1 F(t)\bar{g}(t)\,dt$ **6**
show that this defines an inner product on V.

OR

c) Let $\{x_1, x_2, ...x_n\}$ be an orthogonal set. Then prove that **6**
$\| x_1 + x_2 + ... + x_n \|^2 = \| x_1 \|^2 + \| x_2 \|^2 + ... + \| x_n \|^2$.

d) Find the orthonormal basis of $P_2[-1, 1]$ starting from the basis $\{1, x, x^2\}$ using the inner product defined by $(F, g) = \int_{-1}^{1} F(x)\, g(x)\, dx$ **6**

5. Solve **any six.**

a) Show that the CR equations are satisfied only at z = 0 for $F(z) = |z|^2$. **2**

b) Show that $u = 2x - x^3 + 3xy^2$ is harmonic. **2**

c) Define Basis of vector space. **2**

d) If $v_1, v_2, ..., v_n$ are LI vectors then prove that $v_i \ne 0$ for each i. **2**

e) Show that $T: R^2 \to R^2$ defined by $T(x, y) = (x+1, y+2)$ is not a linear map. **2**

f) Define Isomorphism of vector spaces. **2**

g) Let V be an inner product space over F. Then prove that **2**
$\| \alpha U \| = |\alpha| \| u \|, \forall u \in V, \alpha \in F$.

h) Define orthonormal set. **2**

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