Bachelor of Science (B.Sc.) (Part - II) Fourth Semester

B.Sc. 24112 - Mathematics Paper-II (Classical Mechanics & Statics)

P. Pages : 2 **GUG/W/18/1297**

Time : Three Hours *1010* Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) The moments of a system of forces not in equilibrium acting on a rigid body in one plane about three collinear points A, B, C in the plane are G_1, G_2, G_3. Prove that **6**

$G_1 ; BC + G_2 ; CA + G_3 ; AB = 0$.

b) A uniform rod AB is inclined at an angle 60º to the vertical with one end A resting against a smooth vertical wall. The rod AB is supported by a string attached to a point C of the rod, distance 1 foot from B and also to a ring in the wall vertically above A. If the length of the rod 4 feet, find the position of the ring, reaction of the wall, the inclination and tension of the string. **6**

OR

c) Obtain the cartesian equation of the uniform catenary **6**

$$y = c \operatorname{Cos}\left(\frac{hx}{c}\right)$$

d) A uniform chain of length ℓ is to be suspended from two points A and B in the same horizontal line so that either terminal tension is n times that act the lowest point. Show that the span AB is $\frac{1}{\sqrt{n^2-1}} \log\left[n+\sqrt{n^2-1}\right]$ **6**

UNIT - II

2. a) Prove that the virtual work on a mathematical system by the applied forces and the reversed effective forces is zero. **6**

$$\text{i.e.} \sum_i \left(f_i^{(a)} - \dot{p}_i\right).\delta r_i = 0$$

b) Derive the Lagrange's equation of motion **6**

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0, \; i = 1, 2, 3.....n$$

for conservative system, from D'Alembert's principle.

OR

c) Discuss the motion of a particle in a plane by using polar coordinates. **6**

d) Construct a Lagrangian for a spherical pendulum and then obtain the Lagrange's equations of motion. **6**

UNIT - III

3. a) Prove that the problem of the motion of two masses interacting only with one another always be reduced to a problem of the motion of a single mass. **6**

b) Show that for a central force field F, the path of a particle of mass m is given by **6**

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{h^2u^2} F\left(\frac{1}{u}\right), \; u = \frac{1}{r}$$

OR

c) Prove that the square of the periodic time of the planet is proportional to the cube of the semi-major axis of its elliptic orbit. **6**

d) Show that for a particle moving under a central force such that $V = k r^{n+1}$, the virial theorem reduces to $2\,\bar{T} = (n+1)\,\bar{V}$ **6**

UNIT - IV

4. a) Show that Hamilton's principle is a necessary & sufficient condition for Lagrange's equations. **6**

b) Show that Hamilton's principle can be derived from D'Alembert's principle. **6**

OR

c) Obtain Hamilton's equations from variational principle. **6**

d) Prove that for a single particle system, the least action principle yields. **6**

$$\Delta \int \sqrt{2m\,(H-V)}\; ds = 0, \text{ where } ds = |\,dr\,|$$

5. Solve **any six.**

a) Prove that $S = C \operatorname{Sin} h \frac{x}{c}$ **2**

b) Define virtual work. **2**

c) A bead is sliding on a uniformly rotating wire in a force-free space. Show that the acceleration of the bead is $\ddot{r} = r w^2$, where w is the angular velocity of rotation. **2**

d) Write the Lagrangian and equation of motion for a mass m suspended by a spring of force constant K and allowed to swing vertically. **2**

e) Define the areal velocity. **2**

f) State Kepler's second law of planetary motion. **2**

g) Apply variational principle to find the equation of one dimensional harmonic oscillator. **2**

h) Show that A cyclic coordinate will be absent in Hamiltonian. **2**

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