Bachelor of Science (B.Sc. – II) Fourth Semester

B.Sc. 24111 – Mathematics Paper – I (Abstract Algebra & Differential Equation)

P. Pages : 3 | **GUG/W/18/1296**

Time : Three Hours | Max. Marks : 60

Notes :
1. Solve all the **five** questions.
2. Question 1 to 4 has an alternative solve each question in full or its alternative in full.
3. All questions carry equal marks.

UNIT – I

1. a) Show that in a group G, the mapping $T : G \to G$, given by $T(x) = x^{-1}\ \forall x \in G$ is an automorphism of G if and only if G is abelian. If $x_0 \in G$ with $x_0 \neq x_0^{-1}$ then $T \neq I$. **6**

b) Let G be a group for $g \in G$ define $T_g : G \to G$ by **6**

$T_g x = g^{-1} x g,\ \forall x \in G$

prove that T_g is an automorphism of G.

OR

c) Let G be a group and $g \in G$. Functions $T_g : G \to G,\ t_g : G \to G$ are defined by **6**

$T_g(x) = xg, t_g(x) = gx\ \forall x \in G$,

Prove that

i) $T_g\, t_h = t_h\, T_g\ \ \forall g, h \in G$

ii) If $\theta : G \to G$ is one – one onto such that

$\theta . T_g = T_g . \theta\ \forall g \in G$ then $\theta = t_h$ for some $h \in G$.

d) Prove that, if H, K be subgroups of group G, then HK is a subgroup of G iff HK = KH. **6**

UNIT – II

2. a) If in a ring R, $x^3 = x, \forall x \in R$ then show that R is commutative. **6**

b) If R is a ring with zero element 0, then prove that for all $a, b, c \in R$. **6**

i) $a0 = 0a = 0$

ii) $a(-b) = (-a)b = -(ab)$

iii) $(-a)(-b) = ab$

OR

c) Let the characteristic of the ring R be 2 and let $ab = ba\ \forall a, b \in R$. **6**

Then show that,

$$(a + b)^2 = a^2 + b^2 = (a - b)^2$$

d) Let M be the ring of matrices of order 2 over the field R of real numbers and **6**

$$U = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \middle| \; a, b \in R \right\}$$

$$V = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} \middle| \; a, b \in R \right\}$$

$$W = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \middle| \; a, b, c \in R \right\}$$

be non empty subsets of M.

Show that,

i) U is a right ideal of M but U is not a left ideal.

ii) V is a left ideal of M but not a right ideal.

UNIT – III

3. a) Show that $P_n(1) = 1$ and $P_n(-x) = (-1)^n P_n(x)$ **6**

Hence or otherwise deduce that $P_n(-1) = (-1)^n$.

b) Prove that **6**

$$\int_{-1}^{1} (x^2 - 1) P_n' P_{n+1} \, dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$$

OR

c) Show that **6**

$$J_{1/2}'(x) J_{-1/2}(x) - J_{-1/2}'(x) J_{1/2}(x) = \frac{2}{\pi x}$$

d) Prove that **6**

$$\int_{0}^{x} x^{-n} J_{n+1}(x) \, dx = \frac{1}{2^n n!} - x^{-n} J_n(x)$$

UNIT – IV

4. a) Find the Fourier Series for the function **6**

$f(x) = e^{ax}, \; -L < x < L$

b) Expand the following function in Fourier Series **6**

$f|x) = |x|, \; x \in (-\pi, \pi)$

OR

c) If $f(x)=\begin{cases}0, & -\pi<x<0\\ \sin x, & 0<x<\pi\end{cases}$ **6**

Prove that

$$f(x)=\frac{1}{\pi}+\frac{\sin x}{2}-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2nx}{4n^2-1}$$

d) Find the Fourier Sine series **6**

$$f(x)=\frac{1}{4}-x,\ 0<x<\frac{1}{2}$$
$$=x-\frac{3}{4},\ \frac{1}{2}<x<1$$

5. Solve **any six.**

a) Show that for an abelian group $G,\ I(G)\neq\{I\}$ **2**

b) For $G=S_3$ prove that $G\approx I(G)$. **2**

c) Define the ring. **2**

d) Let R be a ring. Prove that if $a, b\in R$, then **2**

$(a+b)^2=a^2+ab+ba+b^2$

where by x^2 we mean xx.

e) Show that **2**

$$\int_{-1}^{1}x^3 P_3(x)\,dx=\frac{4}{35}$$

f) Show that **2**

$nP_{n-1}=(2n+1)x\ P_n-(n+1)P_{n+1}$

g) Let $f\ \&\ g$ be respectively even and odd functions defined on the real line if $f+g=0$ on R then **2**

Prove that $f(x)=0=g(x)$ on R.

h) Define the periodic function. **2**

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