Bachelor of Science (B.Sc.) Third Semester (Old Course)

B.Sc.23111 / MAT 201 : Mathematics Paper-I
(Advanced Calculus And Group Theory)

P. Pages : 2 **GUG/W/18/1270**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Let G be a group then prove that **6**
i) The identity of group G is unique.
ii) The every $a \in G$ has a unique inverse in G.

b) Prove that a nonempty subset H of the group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$. **6**

OR

c) Prove that any two right cosets of a subgroup are either disjoint or identical. **6**

d) Show that the intersection of two normal subgroup of G is a normal subgroup of G. **6**

UNIT – II

2. a) For $S = \{1, 2, 3 ....9\}$ and $a, b \in A(s)$. Compute $a^{-1} \cdot b\ a$, where a = (5, 7, 9) and b = (1, 2, 3) **6**

b) Prove that $(1, 2, 3, 4, 5, 6, 7, 8)^4 = (1, 5)\ (2, 6)\ (3,7)\ (4, 8)$. **6**

OR

c) Let G be any group, g a fixed element in G. Define $\phi : G \to G$ by $\phi(x) = g\,x\,g^{-1}$. Prove that ϕ is an isomorphism of G into G. **6**

d) If ϕ is a homomorphism of group G into a group G', then prove that **6**
i) $\phi(e) = e'$ and
ii) $\phi(x^{-1}) = [\phi(x)]^{-1}\ \forall\ x \in G$, where e and e' are unit elements of G and G' respectively.

UNIT – III

3. a) Using $\in - \delta$ definition of limit of a function, prove that **6**

$$\lim_{(x,y)\to(1,1)} (x^2 + 2y) = 3$$

b) Prove that the function $f(x, y) = x + y$ is continuous $\forall (x, y) \in R^2$. **6**

OR

c) If $u(x,y)=\dfrac{x^2+y^2}{x+y}$, then prove that $\left(\dfrac{\partial u}{\partial x}-\dfrac{\partial u}{\partial y}\right)^2=4\left(1-\dfrac{\partial u}{\partial x}-\dfrac{\partial u}{\partial y}\right)$. **6**

d) If $z=f(x,y)$ and $x=r\cos\theta, y=r\sin\theta$ then show that **6**

$$\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2=\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2$$

UNIT – IV

4. a) If $xu=yz,\ yv=xz$ and $zw=xy$ then find $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$ **6**

b) Verify Euler's theorem on homogenous function for **6**

$$u=\log\frac{x+y}{x-y}$$

OR

c) Let $f(x,y)$ be defined in an open region D. Suppose that it has a local maximum or local minimum at (x_0,y_0). If the partial derivatives f_x and f_y exists at (x_0,y_0), then prove that $f_x(x_0,\ y_0)=0$ and $f_y(x_0,y_0)=0$. **6**

d) Find by using Lagrange's method of multipliers, the least distance of the origin from the plane $x-2y+2z=9$. **6**

5. Attempt **any six.**

a) If G is a group, then prove that for every $a\in G$. $\left(a^{-1}\right)^{-1}=a$. **2**

b) Define left and right coset of subgroup H of group G. **2**

c) Find the cyclic length of permutation $\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 5 & 2 & 3 & 4 & 1\end{pmatrix}$ **2**

d) If G is group and $\phi: G\to G$ such that $\phi(x)=x^2$. $\forall\ x\in G$, then prove that ϕ is homeomorphism. **2**

e) If $f(x)=x^2,\ x\in R$. then prove that f(x) is continuous at x = 3. **2**

f) If $u=e^x(x\cos y-y\sin y)$ then find $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}$. **2**

g) If $u=f(x,y),\ v=g(x,y)$ are differentiable functions of independent variable x and y then define Jacobians. **2**

h) Define absolute maximum and absolute minimum of function $f(x,y)$ **2**

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