Bachelor of Science (B.Sc.) Second Semester Old

2SMAT 104 - Mathematics Paper-II (Differential Equation and Analysis)

P. Pages : 2 **GUG/W/18/1245**

Time : Three Hours Max. Marks : 60

Notes :
1. Solve all **five** questions.
2. Question No. 1 to 4 has an alternative solve each question in full or its alternative in full.
3. All question carry equal marks.

UNIT – I

1. a) Show that D. E. **6**

$(\sin x \cdot \sin y - xe^{y})dy = (e^{y} + \cos x \cdot \cos y)dx$ is exact and find its solution.

b) Solve the differential equation **6**

$(1+y^2)dx = (\tan^{-1} y - x)\, dy$.

OR

c) Solve the differential equation **6**

$p^3 - 2xyp + 4y^2 = 0$.

d) Find orthogonal trajectory of **6**

$r^n = a^n \cdot \cos n_\theta$

UNIT – II

2. a) Solve the D. E. **6**

$(D^3 - 7D - 6)y = e^{2x} \cdot (1+x)$

b) Solve the D. E. **6**

$y'' + 2y' + y = e^{-x} \cdot \log x$

OR

c) Solve the D. E. **6**

$(1-x^2)y'' - xy' - a^2 y = 0$ of which $y = ce^{a \sin^{-1} x}$ is an integral.

d) Solve the D. E. **6**

$x^2 y'' + 3xy' + 10y = 0$ by changing the independent variable from x to z = log x.

UNIT – III

3. a) Prove that if $\lim \cdot S_n$ exists then if must be unique. **6**

b) Let $X = \{x_n\}$ and $Y = \{y_n\}$ be sequences of real numbers that converges to x & y then prove that sequence X + Y converges to x + y. **6**

OR

c) Prove that every convergent sequence of real numbers is a cauchy sequence. **6**

d) Prove that a sequence $\{s_n\}$ converges if and only if for each $\in > 0$, there exists $M \in N$ such that $|S_m - S_n| < \in$. $\forall m, n \geq M$. **6**

UNIT – IV

4. a) Prove that an infinite series is convergent if and only if its sequence of partial sums is a cauchy sequence. **6**

b) Let $u_n \geq 0$ and $v_n \geq 0 \; \forall \; n$ such that $\lim_{n\to\infty} \frac{u_n}{v_n} = \ell, \quad \ell \neq 0, \infty$. Then prove that Σu_n and Σv_n converges or diverges together. **6**

OR

c) Prove that the series $\sum_{n=1}^{\infty} \frac{n^3 + a}{2^n + a}$ is convergent series by D' Alembert's ratio test. **6**

d) Prove that $\sum\left(\frac{n}{n+1}\right)^n \cdot x^n$, $x > 0$ is divergent for x = 1 and convergent for x < 1. **6**

5. Solve **any six.**

a) Form the differential equation from the equation $y = ACos\,mx + BSinmx$ **2**

b) Solve the linear differential equation **2**

$$y' + y = \frac{1}{1 + e^{2x}}$$

c) Solve the D.E. **2**

$$y'' + y' - 6 = 0.$$

d) Find P. I. of $\frac{1}{D^2 + 1} \cdot \sin 2x$ **2**

e) Define the limit of sequence at infinity. **2**

f) Show that $\lim_{n\to\infty} n^{y_n} = 1$ by Cauchy formula. **2**

g) Show that if Σu_n converges then $\Sigma k u_n$ converges for $k \in R$. **2**

h) Test the convergence of series $\sum_{n=1}^{\infty} \frac{1}{n}$. **2**

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