Bachelor of Science (B.Sc.)-I Second Semester Old

2SMAT 103 - Mathematics Paper-I
(Vector Calculus, Geometry & Difference Equation)

P. Pages : 2 | GUG/W/18/1244

Time : Three Hours

* 0 9 5 7 *

Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. Question 1 to 4 has an alternative. Solve each question in full or its alternative in full.
3. All questions carries equal marks.

UNIT - I

1. a) Prove that $\bar{a} \times \left(\bar{b} \times \bar{c}\right) = \left(\bar{a}.\,\bar{c}\right)\bar{b} - \left(\bar{a}.\bar{b}\right)\bar{c}.$ **6**

b) If $\bar{a} = \bar{i} - \bar{j} - \bar{k},\ \bar{b} = \bar{j} + \bar{k},\ \bar{c} = \bar{i} - \bar{j}$ **6**

find i) $\left|\left(\bar{a} \times \bar{b}\right) \times \bar{c}\right|$ ii) $\bar{a}.\left(\bar{b} \times \bar{c}\right)$

OR

c) If $\bar{V} = \bar{w} \times \bar{r}$, prove that $\bar{w} = \frac{1}{2}$ curl $\bar{V}$, where $\bar{w}$ is a constant vector. **6**

d) If $\phi = x^3 + y^3 + z^3 - 3xyz$. Find div grad$\phi$ and curl grad ϕ. **6**

UNIT - II

2. a) Prove that **6**

i) $f(x, y) \geq 0$ on $R \Rightarrow \iint_R f(x, y)\,dA \geq 0$

ii) $f(x, y) \leq g(x, y) \Rightarrow \iint_R f(x, y)\,dA \leq \iint_R g(x, y)\,dA$

b) Evalueat $\int_0^{\pi/2} \int_0^{2a\cos\theta} r^2 \sin\theta\, dr\, d\theta$ **6**

OR

c) Evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration. **6**

d) Evaluate by changing to polar co-ordinates. **6**

$\iint_R \frac{x^2}{\sqrt{x^2 + y^2}}\,dx\,dy$ where R is the region defined by $0 \leq y \leq x,\ 0 \leq x \leq a$

UNIT - III

3. a) Find the co-ordinates of the centre and the radius of the circle **6**

$x+2y+2z=15;\ \ x^2+y^2+z^2-2y-4z=11.$

b) Find the equation of a sphere for which the circle **6**

$x^2+y^2+z^2+7y-2z+2=0;\ 2x+3y+4z=8$ is a great circle.

OR

c) Find the equation of the cone whose vertex is the point (1,2,3) and the guiding curve is the circle $x^2+y^2+z^2=4;\ \ x+y+z=1.$ **6**

d) Prove that every homogeneous equations of second degree in x , y and z represents a cone whose vertex is at the origin. **6**

UNIT - IV

4. a) From the equation $y_n = A.3^n + B.5^n$, derive a difference equation by eliminating arbitrary constants A and B. **6**

b) Solve $y_{k+2} - y_{k+1} + y_k = 0,\ k \ge 0,$ Given that $y_0 = 1,\ y_1 = \frac{1+\sqrt{3}}{2}$ **6**

OR

c) Solve difference equation. **6**

$y_{n+2} + 3y_{n+1} + 2y_n = 2^n$, given $y_0 = 0,\ y_1 = 1$

d) Solve $y_{n+2} - 2y_{n+1} + y_n = n^2.\ 2^n$ **6**

5. Solve **any six.**

a) Find volume of a parallelepiped with $\bar{a} = (1,0,2),\ \bar{b} = (0,3,1),\ \bar{c} = (2,2,0)$ as edge vectors. **2**

b) If $\bar{f} = \sin t\ \bar{i} + \cos t\ \bar{j},\ \bar{g} = t\ \bar{i} + t^2\bar{j} - t^3\bar{k}$ **2**

find $\frac{d}{dt}\left(\bar{f}\cdot\bar{g}\right)$ at $t = 0$

c) Prove that $\left| \iint_R f(x,y)\,dA \right| \le \iint_R |f(x,y)|\,dA$ **2**

Where f (x, y) is continuous function.

d) Evaluate $\int_0^1 \int_1^3 xy^2\ dy\ dx$ **2**

e) Find equation of sphere with centre (1, -1, 2) and radius 3. **2**

f) Find the equation of cone with vertex at the origin and direction cosines of its generator satisfying the relation $3l^2 - 4m^2 + 5n^2 = 0$. **2**

g) Solve $y_{n+3} - 2y_{n+2} - 5\ y_{n+1} + 6y_n = 0$ **2**

h) Prove that if $V(n) = a^n$ then particular integral is $\frac{1}{P(E)}a^n = \frac{1}{P(a)}a^n$ provided $P(a) \ne 0$. **2**

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