Bachelor of Science (B.Sc. – I) First Semester (Old Course)

MAT-102 – Mathematics Paper – II (Differential and Integral Calculus)

P. Pages : 2 **GUG/W/18/1215**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve **all five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Let $f(x)$ and $g(x)$ be defined at all points of an interval $[a, b]$ except possibly at $x_0 \in [a, b]$. If $\lim_{x \to x_0} f(x) = A$, $\lim_{x \to x_0} g(x) = B$ then prove that $\lim_{x \to x_0} [f(x) + g(x)] = A + B$ **6**

b) Prove that if $f(x) = \sqrt{x-2}$ for $2 \le x \le 4$ then $f(x)$ is continuous in the interval $[2, 4]$. **6**

OR

c) If a real function f defined on $[a, b]$ is (i) continuous on $[a, b]$ (ii) differentiable on $[a, b]$ then prove that there is atleast one point $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. **6**

d) Obtain Maclaurin's series for $f(x) = \sin x$. **6**

UNIT – II

2. a) If $y = e^{ax} \sin(bx + c)$ then prove that $y_n = r^n e^{ax} \sin(bx + c + n\theta)$ where $r = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1} \frac{b}{a}$. **6**

b) If $y = 0.\cos(\log x) + b.\sin(\log x)$ then show that **6**

i) $x^2 y_2 + x y_1 + y = 0$

ii) $x^2 y_{n+2} + (2n+1) x y_{n+1} + (n^2 + 1) y_n = 0$

OR

c) Evaluate $\lim_{x \to 0} \frac{x - \sin x}{x^3}$ **6**

d) Evaluate $\lim_{x \to 0} \left(\cot x - \frac{1}{x} \right)$ **6**

UNIT – III

3. a) Evaluate : $\int \frac{x^3 + 1}{\sqrt{x^2 + 3}}$ **6**

b) Evaluate $\int \frac{x^2 + 2x + 3}{\sqrt{x^2 + x + 1}}$ **6**

OR

c) If $I_n = \int \sin^n x.dx$, then prove that **6**

$$I_n = -\frac{1}{n}\sin^{n-1}x.\cos x + \frac{n-1}{n}I_{n-2} \text{-----}$$

d) Prove that **6**

$$\int \sec^n x\,dx = \frac{\sec^{n-2}x\tan x}{x-1} + \frac{n-2}{n-1}\int \sec^{n-2}x.dx$$

UNIT – IV

4. a) Prove that **6**

$$\beta(m,n) = \frac{\lceil m \cdot \lceil n}{\lceil m+n}$$

b) Prove that **6**

$$\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta\cos^{2n-1}\theta\,d\theta$$

OR

c) Prove that **6**

i) $\lceil n+1 = n\lceil n$

ii) $\int_0^{\infty} x^{n-1}\cdot e^{-kx}dx = \frac{\lceil n}{k^n}$ where $n, k > 0$ is constants.

d) Prove that **6**

$$\int_0^{\pi/2}\sqrt{\tan\theta\cdot d\theta} = \frac{\pi}{\sqrt{2}}$$

5. Solve **any six.**

a) Evaluate $\lim_{x\to 3}(2x^3 - 3x^2 + 7x - 11)$. **2**

b) Show that $f(x) = \frac{1}{1-e^{1/x}}$ has a simple discontinuity at $x = 0$. **2**

c) If $y = \sin(ax+b)$ then prove that **2**

$$y_n = a^n \cdot \sin\left(ax + b + \frac{1}{2}n\pi\right).$$

d) If $y = A\cdot\sin mx + B\cdot\cos mx$ then prove that $y_2 + m^2y = 0$. **2**

e) Evaluate $\int \frac{dx}{\sqrt{x^2+2x+5}}$ **2**

f) Evaluate $\int \cos^5 x\cdot dx$ **2**

g) Show that $\int_0^{\infty} e^{-x^2}\cdot dx = \frac{\sqrt{\pi}}{2}$ **2**

h) Evaluate $\beta\left(\frac{3}{2}, 3\right)$ **2**

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