Bachelor of Science (B.Sc.) First Semester Old Course

MAT 101 - Mathematics Paper – I – (Algebra and Trigonometry)

P. Pages : 2 | **GUG/W/18/1214**

Time : Three Hours | Max. Marks : 60

Notes :
1. Solve **all five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that $\dfrac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}=\sin\theta+i\cos\theta$ **6**

Hence prove that

$$\left(1+\sin\frac{\pi}{5}+i\cos\frac{\pi}{5}\right)^5+\left(1+\sin\frac{\pi}{5}-i\cos\frac{\pi}{5}\right)^5=0$$

b) Prove that **6**

$$\sin 6\theta=6\cos^5\theta\sin\theta-20\cos^3\theta\sin^3\theta+6\cos\theta\sin^5\theta$$

OR

c) Separate into real and imaginary parts of $\tan h(x+iy)$. **6**

d) If $\sin(\theta+i\phi)=\cos\alpha+i\sin\alpha$. Prove that $\cos^2\theta=\pm\sin\alpha$. **6**

UNIT – II

2. a) For the matrix $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 3\\ 0 & -1 & -1\end{bmatrix}$, find non singular matrices P and Q such that PAQ is in the normal form. **6**

b) Solve the linear equations $2x+3y-z=0,\ x-y+2z=5,\ 3x+y-z=1$ by matrix method. **6**

OR

c) Show that if B be an invertible matrix of the same order as A, then the matrices A and $B^{-1}\cdot A\cdot B$ have the same characteristic roots. **6**

d) Find the characteristic equation of the matrix $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ and verify Cayley – Hamilton theorem for the matrix. Hence or otherwise express $A^5-4A^4-7A^3+11A^2-A-10I$ as linear polynomial in A. **6**

UNIT – III

3. a) Prove that every equation of degree n has n roots and no more. **6**

b) Solve the following equation $6x^3 - 11x^2 - 3x + 2 = 0$, the roots being in harmonic progression. **6**

OR

c) Show that the same transformation removes both 2nd and 4th terms of the equation $x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$. **6**

d) Solve by Cardon's method $x^3 - 21x = 344$. **6**

UNIT – IV

4. a) Use the first principle of induction to prove the following **6**
$$1^2 + 2^2 + 3^3 + ... + n^2 = \frac{1}{6} n(n+1)(2n+1), \ \forall n \in N$$

b) Let $\frac{a}{b}$ and $\frac{c}{d}$ be fractions in lowest terms so that $(a, b) = (c, d) = 1$. **6**
Prove that if their sum is an integer, then $|b| = |d|$.

OR

c) Find the gcd of 275 and 200 and express it in the form 275m + 200n. **6**

d) Find positive integers a and b satisfying the equations (a, b) = 10 and [a, b] = 100 simultaneously. Find all solutions. **6**

5. Solve **any six.**

a) State De Moivre's Theorem. **2**

b) Separate $\sin(x + iy)$ into real and imaginary parts. **2**

c) Define transpose of a matrix. **2**

d) Show that if λ is the eigenvalue of a non singular matrix A, then λ^{-1} is the eigenvalue of A^{-1}. **2**

e) Find the nature of the roots of the equation $3x^4 + 12x^2 + 5x - 4 = 0$. **2**

f) Determine the values of $\Sigma\alpha^2$ for the cubic equation $x^3 + px^2 + qx + r = 0$ whose roots are α, β, γ. **2**

g) Prove that $(a, a + k) | k$ for all integers a, k not both zero. **2**

h) Evaluate $(n, n+1)$, where $n \in N$. **2**

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