Bachelor of Science (B.Sc.) (CBCS Pattern) Third Semester CBCS

USMT-06 -Mathematics Paper-II - Set Theory and Laplace Transform

P. Pages : 2 **GUG/W/18/11613**

Time : Three Hours * 3 8 6 8 * Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that $A\times(B\cup C) = (A\times B)\cup(A\times C)$ **6**

b) Show that a relation R is symmetric if and only if $R^{-1} = R$. **6**

OR

c) Let $a, b, x, y \in R$ such that $b > 0,\ y > 0$ and $\frac{a}{b} < \frac{x}{y}$ then show that $ay < bx$. **6**

d) Prove that every infinite subset of a countable set is countable. **6**

UNIT – II

2. a) Let $A, B \in \tilde{P}(U)$. Then prove that $\alpha \le \beta \Rightarrow \beta_A \subseteq {}^{\alpha}A$ and ${}^{\beta+}A \subseteq {}^{\alpha+}A,\ \forall \alpha, \beta \in [0,1]$. **6**

b) For $U = \{1, 2, 3, 4, 5\}$, $\tilde{A} = \frac{0.1}{1} + \frac{0.3}{2} + \frac{1}{5}$ and $\tilde{B} = \frac{0.4}{2} + \frac{0.2}{3}$ find $\tilde{A} + \tilde{B}$ **6**

OR

c) For any $\tilde{A} \in P(U)$, Prove that ${}^{\alpha}\tilde{A} = \bigcap_{\beta<\alpha} {}^{\beta}\tilde{A}$ **6**

d) Let $\tilde{A}, \tilde{B} \in P(U)$ then prove that for all $\alpha \in [0,1],\ \tilde{A} \subseteq \tilde{B} \Leftrightarrow {}^{\alpha}\tilde{A} \subseteq {}^{\alpha}\tilde{B}$ **6**

UNIT – III

3. a) If $c_1, c_2, ..., c_n$ are any constants and $f_1(t), f_2(t), ..., f_n(t)$ are functions whose Laplace transforms exists then prove that **6**
$L[c_1 f_1(t) + c_2 f_2(t) + ... + c_n f_n(t)] = c_1 L[f_1(t)] + c_2 L[f_2(t)] + ... + c_n L[f_n(t)]$
have find $L[\sin at]$.

b) Find Laplace transform of function $\left(2e^t \sin 4t \cos 2t\right)$. **6**

OR

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c) Let f(t) be a periodic function of period T > 0, then prove that **6**

$$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt$$

d) Find the Laplace transform of the function. **6**

$$f(t) = \begin{cases} t & 0 < t < a \\ 2a - t & a < t < 2a \end{cases} \quad \text{where } f(t+2a) = f(t)$$

UNIT – IV

4. a) Find the inverse Laplace transform of **6**

i) $\frac{s^2 - 3s + 4}{s^3}$ ii) $\frac{(s^2-1)^2}{s^5}$

b) Verify the convolution theorem for $f_1(t) = t,\ f_2(t) = \cosh t$. **6**

OR

c) Solve $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 4t + e^{3t}$ when $x(0) = 1$, and $x'(0) = -1$. **6**

d) Find the solution by Laplace transform of $(D^2 + n^2)x = a\sin(nt + \alpha),\ x = Dx = 0$ at $t = 0$. **6**

5. Attempt **any six.**

a) Let R be a relations from A={2, 4, 9} to B = {2, 3} defined by $R = \{(a,b) / a \in A,\ b \in B, a \text{ is divisible by } b\}$ Find the relation R. **2**

b) Prove that $xy = zy \Rightarrow x = z\ \forall x, y, z \in R,\ y \neq 0$. **2**

c) Define scaler cardinality and relative cardinality of a fuzzy set F. **2**

d) Define union of two fuzzy sets. **2**

e) Find $L[e^{-at}]$ by using definition of Laplace transform. **2**

f) Find $L[\sin(\omega t + \alpha)]$ **2**

g) Find $L^{-1}\left(\frac{s+3}{s^2+1}\right)$ **2**

h) If $L^{-1}[F(s)] = f(t)$, then prove that $L^{-1}[F(s-a)] = e^{at} f(t)$ **2**

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