Bachelor of Science (B.Sc.) (CBCS Pattern) Third Semester CBCS

USMT-05 -Mathematics Paper-I : Real Analysis

P. Pages : 2 **GUG/W/18/11612**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT - I

1. a) Prove that if $\lim S_n$ exists, it must be unique. **6**

b) Find the limit of sequence $\langle S_n \rangle$, where $\langle S_n \rangle = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + ... \frac{1}{(n+n)^2}$ **6**

OR

c) Prove that every convergent sequence of real numbers is a Cauchy sequence. **6**

d) Show that **6**

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + ... + \frac{1}{\sqrt{n^2+n}}\right] = 1$$

UNIT – II

2. a) Prove that the series Σx_n converges if and only if for every $\in > 0\ \exists$ a number $M(\in) \in N$ such that **6**

$m \geq n \geq M \Rightarrow /x_{n+1} + x_{n+2} + ... + x_m/ < \in$

b) Prove that a geometric series $\sum_{n=1}^{\infty} x^{n-1}$ converges to $\frac{1}{1-x}$ for 0 < x < 1 and diverges for $x \geq 1$. **6**

OR

c) Let $x_n \geq 0,\ y_n \geq 0\ \forall n \in N$ and let $\exists m \in N$ such that **6**

$x_n \leq k\, y_n\ \forall n \geq m,\ k > 0,$ then show that

i) Σy_n converges $\Rightarrow \Sigma x_n$ converges

ii) Σx_n diverges $\Rightarrow \Sigma y_n$ diverges

d) Define Alternating series and test the convergence of series **6**

$\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + ...$

UNIT – III

3. a) Show that $d(x, y) = |x - y|\ \forall x, y \in R$ defines a metric on R. **6**

b) Show that every neighbourhood is an open set. **6**

OR

c) Let $\{A_\alpha\}$ be a finite or infinite collection of sets A_α. Then prove that **6**

$$\left[\bigcup_{\alpha} A_\alpha\right]^C = \bigcap_{\alpha} A_\alpha{}^C$$

d) Prove that every convergent sequence in a metric space is a Cauchy sequence. **6**

UNIT – IV

4. a) Show that any constant function defined on a bounded closed interval is integrable. **6**

b) If f is a bounded and integrable over [a, b] and M, m are bounds of f over [a, b], then prove that **6**

$$m(b-a) \le \int_a^b f(x)dx \le m(b-a)$$

OR

c) Let a function f be continuous and non-negative on the interval [a, b]. Prove that **6**

$F(x) = \int_a^x f(t)dt,\ x \in [a, b]$ is monotonic non-decreasing in [a, b].

d) Prove that if f be a bounded and integrable function defined on [a, b] with m, M as infimum, supremum respectively, then there exists a number μ between m and M such that **6**

$$\int_a^b f(x)dx = \mu(b-a)$$

5. Solve **any six.**

a) Prove that $\lim_{n\to\infty} \frac{1}{n^2} = 0$ **2**

b) Evaluate $\lim S_n$ for the sequence **2**

$S_n = \sqrt{n+a} - \sqrt{n+b},\ a \exists b$

c) Test the convergence of series by using P-series test **2**

$\sum \frac{1}{\sqrt{n}} = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ...$

d) Test for the convergence of the series **2**

$$\sum \left(\frac{n}{n+1}\right)^{n^2}$$

e) Define metric. **2**

f) Define open set. **2**

g) Define Darboux's upper & lower sums. **2**

h) Prove that **2**

$m(b-a) \le L(p, f) \le U(p, f) \le M(b-a)$

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