Bachelor of Science (B.Sc.)-I (CBCS Pattern) Second Semester CBCS

USMT-03 - Mathematics Paper-I
(Ordinary Differential Equations and Difference Equations)

P. Pages : 2 **GUG/W/18/11586**

Time : Three Hours * 3 6 6 1 * Max. Marks : 60

Notes : 1. Solve all **five** question.
2. Question 1 to 4 has an alternative solve each question in full or it's alternative in full.
3. All question carry equal marks.

UNIT - I

1. a) Prove that necessary and sufficient condition for the differential equation Mdx + Ndy = 0 to be exact is that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ **6**

b) Solve $\left(3x^2 y^4 + 2xy\right) dx + \left(2x^3 y^3 - x^2\right) dy = 0$ **6**

OR

c) Find the orthogonal trajectories of the family of curves **6**
$\frac{x^2}{a^2} + \frac{y^2}{b^2 + k} = 1,$ Where K is a parameter.

d) Solve $x \frac{dy}{dx} + y = y^2 \log x$ **6**

UNIT - II

2. a) Solve : $y'' - 4y' + 4y = e^{2x} + \sin 2x.$ **6**

b) Solve : $\left(D^4 + n^4\right) y = 0.$ **6**

OR

c) Solve : $\frac{dx}{dt} + 5x - 2y = t, \frac{dy}{dt} + 2x + y = 0$ **6**

d) Solve $\left(D^2 - 4D + 4\right) y = 8x^2 e^{2x} \sin 2x.$ **6**

UNIT - III

3. a) Solve $x^2 \frac{d^2y}{dx^2} - 8x \frac{dy}{dx} + 8y = \log x$ **6**

b) Solve $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$ **6**

OR

c) If $y_1(x)$ and $y_2(x)$ are any two solutions of the second order homogeneous differential equation $y''+P(x)y'+Q(x)y=0$ on $[a,b]$, then prove that their Wronskian $W=W(y_1,y_2)$ is given by $W=ce^{-\int pdx}$ and it is either identically zero or never zero on [a,b]. **6**

d) Solve : $y''-y=\frac{2}{1+e^x}$,by the method of variation of parameters. **6**

UNIT - IV

4. a) Show that $u_x=2^x\left(C_1\cos\frac{2\pi x}{3}+C_2\sin\frac{2\pi x}{3}\right)$ is a solution of $u_{x+2}+2u_{x+1}+4u_x=0$ Where C_1 & C_2 are arbitrary periodic functions of period unity. **6**

b) Solve $u_{x+1}-\frac{1}{x}ux=0,\ x>0$ **6**

OR

c) Solve : $y_{n+3}-2y_{n+2}-5y_{n+1}+6y_n=0$ **6**

d) Solve : $y_{k+1}+\frac{1}{4}y_k=\left(\frac{1}{4}\right)^k,\ k\geq 0,\ y(0)=1.$ **6**

5. Solve **any six.**

a) Solve $e^y dx+\left(1+xe^y\right)dy=0$ **2**

b) Define Bernoulli's differential equation. **2**

c) Solve : $\frac{d^3y}{dx^3}-13\frac{dy}{dx}+12y=0$ **2**

d) Find P. I. of $\left(D^2-2D+1\right)y=xe^x\cos x$ **2**

e) Find the complementary function of $\left(x^2D^2-xD+4\right)y=\cos(\ln x)$ **2**

f) Define Wronskian W of any two solutions y_1 & y_2 of the differential equation. **2**

g) Define order of difference equation. **2**

h) Find the complementary function of $u_{n+2}+4u_{n+1}+3u_n=2^n$. **2**

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