Bachelor of Science (B.Sc.) (CBCS Pattern) First Semester

USMT-02 - Mathematics Paper-II (Differential Calculus and Trigonometry)

P. Pages : 2 **GUG/W/18/11557**

Time : Three Hours Max. Marks : 60

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Notes : 1. Solve **all five** questions.
2. Each question carries equal marks.

UNIT - I

1. a) Using $\in - \delta$ definition of Limit prove that $\lim\limits_{(x,y)\to(1,1)} \left(x^2 + 2y\right) = 3$. **6**

b) Let $z = f(x, y) = \dfrac{y\left(x^2 + y^2\right)}{y^2 + \left(x^2 + y^2\right)^2}$ **6**

Show that f has limit 0 as $(x, y) \to (0, 0)$ on a ray x = at, y = bt, but f doesn't have limit 0 as $(x, y) \to (0, 0)$ along $y = x^2$.

OR

c) If $u = \dfrac{e^{x+y+z}}{e^x + e^y + e^z}$, show that $u_x + u_y + u_z = 2u$ where $u_x = \dfrac{\partial u}{\partial x}$ etc. **6**

d) If $u = F(x - y,\ y - z,\ z - x)$, then prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. **6**

UNIT - II

2. a) Verify Euler's theorem as homogeneous functions for $u = \mathrm{Log}\left(\dfrac{x+y}{x-y}\right)$. **6**

b) If $x = r\sin\theta\cos\phi$ $y = r\sin\theta\sin\phi$. $z = r\cos\theta$, find $\dfrac{\partial(x,\ y,\ z)}{\partial(r,\ \theta,\ \phi)}$. **6**

OR

c) Expand $x^3 - 2xy^2$ in Taylor's series about the point (1, -1). **6**

d) Find by using Lagrange's method of multipliers, the least distance of the origin from the plane $x - 2y + 2z = 9$. **6**

UNIT - III

3. a) Find the asymptotes of the curve. **6**
$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 = 1$.

b) Find the curvature of the cycloid $x = \theta - \sin\theta,\ y = 1 - \cos\theta$ at the highest point of an arc. **6**

OR

c) Trace the curve: $a^2 y^2 = x^2\left(a^2 - y^2\right)$. **6**

d) Find the asymptotes of the curve $3x^2 + 2x^2y - 7xy^2 + 2y^3 - 14xy + 7y^2 + 4x + 5y = 0$. **6**

UNIT - IV

4. a) If n is a positive or negative integer, prove that **6**
$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

b) Show that **6**
$$(a+ib)^{m/n} + (a-ib)^{m/n} = 2\left(a^2+b^2\right)^{m/2n} \cdot \cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right).$$

OR

c) Prove that $\tan h^{-1} x = \frac{1}{2}\,\text{Log}\,\frac{1+x}{1-x}$. **6**

d) Show that **6**
$$\text{Log}(1+i) = \frac{1}{2}\log 2 + i\pi\left(2n + \frac{1}{4}\right).$$

5. Attempt **any six.**

a) Define $E - \delta$ definition of continuity of function f (x, y). **2**

b) If $u = x^2 + y^2$ where $x = at^2,\ y = 2at$, find $\frac{du}{dt}$. **2**

c) State Euler's theorem for homogeneous function of two variables. **2**

d) If $x = r\cos\theta,\ y = r\sin\theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$ and hence find $\frac{\partial(r,\theta)}{\partial(x,y)}$. **2**

e) Define Asymptote of a curve. **2**

f) Write the formula of radius of curvature of a curve in the cartesian form. **2**

g) Express $\left(i + i\sqrt{3}\right)$ in polar form. **2**

h) Separate cos h (x+iy) into real and imaginary parts. **2**

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