Bachelor of Science (B.Sc.) (CBCS Pattern) First Semester

USMT-01 – Mathematics Paper – I – (Differential and Integral Calculus)

P. Pages : 3 | GUG/W/18/11556

Time : Three Hours | Max. Marks : 60

Notes : 1. Solve **all five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Let $f(x)$ and $g(x)$ be defined at all points of an interval $[a, b]$ except possibly at $x_0 \in [a, b]$ if $\lim_{x \to x_0} f(x) = A$, $\lim_{x \to x_0} g(x) = B$ Then prove that **6**

$$\lim_{x \to x_0} [f(x) + g(x)] = \lim_{x \to x_0} f(x) + \lim_{x \to x_0} g(x)$$
$$= A + B$$

b) Discuss the continuity of the function **6**

$$f(x) = (x - a)\sin\frac{1}{x - a}, \quad x \neq a$$
$$= 0 \quad , \quad x = a$$

OR

c) If f (x) is differentiable at $x = x_0$, then prove that it is continuous at x_0. **6**

d) If $y = (x^2 - 1)^n$, prove that $(x^2 - 1) y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$. **6**

UNIT – II

2. a) If a real valued function f defined on [a, b] is (i) continuous on [a,b], ii) differentiable on (a, b) and iii) f(a) = f(b) then prove that there is at least one point $c \in (a, b)$ such that $f'(c) = 0$ **6**

b) Verify Lagrange's mean value theorem for $f(x) = \log x$ in $[1, e]$. **6**

OR

c) Obtain Maclaurin's series for $f(x) = \sin x$. **6**

d) Obtain Taylor's series for $2x^3 + 7x^2 + x - 1$ in powers of $(x - 2)$. **6**

UNIT – III

3. a) Prove that **6**

i) $\Gamma n = 2\int_0^\infty e^{-t^2} t^{2n-1} dt$, ii) $\int_0^\infty e^{-x^{1/n}} dx = n\Gamma n$.

b) Show that **6**

$$\beta(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$$

OR

c) Evaluate **6**

$$\lim_{x\to\infty}\frac{\log\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$$

d) Evaluate $\lim_{x\to 0}(\cot x)^{\sin 2x}$. **6**

UNIT – IV

4. a) Let $f(x,y)$, $g(x,y)$ are continuous functions defined on a region D of area A. Then prove that **6**

i) $\iint_D C f(x,y)\,dA = C\iint_D f(x,y)\,dA$ where C is a constant.

ii) $\iint_D [f(x,y)+g(x,y)]\,dA=\iint_D f(x,y)\,dA+\iint_D g(x,y)\,dA$

b) Evaluate : **6**

$$\int_0^\infty dx\int_0^1\frac{dy}{1+x^2y}.$$

OR

c) Evaluate the following integral by changing the order of integration **6**

$$\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$$

d) Evaluate by changing to polar co-ordinates $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$. **6**

Hence show that $\int_0^\infty e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$.

5. Solve **any six.**

a) By $\in - \delta$ definition of limit prove that $\lim_{x\to 2} f(x) = 7$, where $f(x) = 2x+3$, and $x \in [0,5]$ **2**

b) If $y = A\sin mx + B\cos mx$, then prove that $y_2 + m^2 y = 0$ where $y_2 = \frac{d^2y}{dx^2}$. **2**

c) State Lagrange's mean value theorem. **2**

d) Write power series in x and $(x-a)$. **2**

e) Prove that $\int_0^{\pi/2} \sin^2\theta \cos^4\theta \, d\theta = \frac{\pi}{32}$ **2**

f) Evaluate $\lim_{x\to 0} \frac{x - \sin x}{x^3}$ **2**

g) Evaluate $\int_0^1 \int_1^3 xy^2 \, dy \, dx$ **2**

h) Evaluate $\int_0^{\log 8} \int_0^{\log y} e^{x+y} dx \, dy$. **2**

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