B.E - Bachelor of Engineering (CBCS PATTERN) Second Semester

2BEAB01 - Applied Mathematics - II

P. Pages : 2 | **GUG/W/18/11471**

Time : Three Hours | Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of Non-programmable calculator is permitted.

1. a) Solve $\left(x^3-x\right)\frac{dy}{dx}-\left(3x^2-1\right)y=x^5-2x^3+x$. **4**

b) Solve $\frac{dy}{dx}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$. **4**

c) Solve by method of variation of parameter. **8**

$$\frac{d^2y}{dx^2}-y=e^{-x}\sin\left(e^{-x}\right)+\cos\left(e^{-x}\right).$$

OR

2. a) Solve $\left(xy^2-e^{1/x^3}\right)dx-x^2y\,dy=0$. **4**

b) Solve $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xy\,e^{xy^2}-3y^2\right)dy=0$. **4**

c) Solve $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+13y=e^{2x}\sin x+4^x$. **8**

3. a) Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+4y=\log x\cos(\log x)$. **8**

b) Solve $\frac{d^2y}{dx^2}=2\left(y^3+y\right)$ under the condition $y=0,\ \frac{dy}{dx}=1$, when x= 0. **8**

OR

4. a) Solve $(x+3)^2\frac{d^2y}{dx^2}-4(x+3)\frac{dy}{dx}+6y=\log(x+3)$. **8**

b) Solve **8**

$$\frac{dx}{dt}+2x-3y=t;\ \frac{dy}{dt}-3x+2y=e^{2t}.$$

5. a) Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$. **8**

b) Evaluate $\int_{0}^{a}\int_{y}^{a}\frac{x^2}{\sqrt{x^2+y^2}}\,dx\,dy$ **8**

by change of order of integration.

OR

6. a) Find by double integration the area lying between the parabola $y = x^2 - 6x + 3$ and the line $y = 2x - 9$. **8**

b) Find the centre of gravity of the area between $y = 6x - x^2$ and y = x. **8**

7. a) Find the angle between the tangents to the curve $\bar{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$ at the points $t = \pm 1$. **4**

b) Find the tangential and normal components of acceleration at any time t of a particle whose position at time t is given by **5**

$\bar{r} = t^2\hat{i} - t^3\hat{j} + t^4\hat{k}$ at $t = 1$.

c) Find the directional derivatives of **7**

$\phi = \left(x^2 + y^2 + z^2\right)^{-1/2}$ at the point (3, 1, 2) in the direction of the vector $yz\hat{i} + zx\hat{j} + xy\hat{k}$.

OR

8. a) The position vector of a point at time t is given by $\bar{r} = e^t\left(\cos t\,\hat{i} + \sin t\,\hat{j}\right)$ show that **8**

i) $\bar{a} = 2(\bar{v} - \bar{r})$ where $\bar{a}$, $\bar{v}$ are acceleration and velocity of the particle.

ii) The angle between the radius vector and acceleration is constant.

b) Find the values of constant a, b, c so that the directional derivatives of $\phi = ax^2 + by^2 + cz^2$ (1, 1, 2) has a maximum magnitude 4 in the direction parallel to x=axis. **8**

9. a) Prove that:

i) Div $\left(\frac{\bar{a} \times \bar{r}}{r^3}\right) = 0$. **4**

ii) Curl $\left(\frac{\bar{a} \times \bar{r}}{r^3}\right) = \frac{-\bar{a}}{r^3} + \frac{3(\bar{a} \cdot \bar{r})}{r^5}\bar{r}$ **4**

where $\bar{a}$ is constant vector.

b) A vector field $\bar{F}$ is given by $\bar{F} = \left(6xy + z^3\right)\hat{i} + \left(3x^2 - z\right)\hat{j} + \left(3xz^2 - y\right)\hat{k}$. **8**

Prove that it is irrotational and hence find its scalar potential.

OR

10. a) Evaluate $\iint \bar{F} \cdot \hat{n}\, ds$ by Gauss Divergence theorems and S is the surface of cylinder $x^2 + y^2 = 4$, $z = 0$, $z = 3$ and $\bar{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$. **8**

b) Apply Stoke's theorem to evaluate **8**

$$\oint_C [(x + y)dx + (2x - z)dy + (y + z)dz]$$

where C is boundary of the triangle with vertices (2, 0; 0) (0, 2, 0) and (0, 0, 2).

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