B.E . Bachelor of Engineering (CBCS Pattern) First Semester

1BEAB01 - Applied Mathematics-I

P. Pages : 3 **GUG/W/18/11465**

Time : Three Hours * 3 5 5 0 * Max. Marks : 80

Notes : 1. All questions carry equal marks.
3. Use of Non-Programmable calculator is permitted.

1. a) Find n^{th} differential coefficient of

i) $\sin x \cdot \sin 2x \cdot \sin 3x$. **4**

ii) $\dfrac{1}{\left(6x^2+11x+3\right)}$ **5**

b) If $\cos^{-1}\left(\dfrac{y}{b}\right)=\log\left(\dfrac{x}{n}\right)^n$ then **7**

show that

$x^2y_{n+2}+(2n+1)xy_{n+1}+2x^2y_n=0.$

OR

2. a) Use Taylor's series expansion to evaluate sin 60º 30' correct upto four decimal places. **4**

b) Evaluate. **4**

$\lim\limits_{x\to 0}\dfrac{\sin x^2-\sin^2 x}{x^4}$

c) **8**

If $y=e^{a\sin^{-1}x}$ then show that

$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-\left(n^2+a^2\right)y_n=0$

Hence find $y_n(0)$.

3. a) **8**

If $\theta=t^n e^{-\frac{r^2}{4t}}$ then find the value of n such that

$\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial\theta}{\partial r}\right)=\dfrac{\partial\theta}{\partial t}$.

b) If $u = \text{cosec}^{-1}\sqrt{\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}$ then **8**

show that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\cdot\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{144}\left[13+\tan^2 u\right].$$

OR

4. a) If $x = e^r\cos\theta, y = e^r\sin\theta$ then show that **8**

$$e^{-2r}\left[\left(\frac{\partial z}{\partial r}\right)^2+\left(\frac{\partial z}{\partial \theta}\right)^2\right]=\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2.$$

b) If $f(x,y)=\frac{1}{x^2}+\frac{1}{xy}-\frac{\log y-\log x}{x^2+y^2}$ **8**

then show that

$$x^2\frac{\partial^2 f}{\partial x^2}+2xy\frac{\partial^2 f}{\partial x\cdot\partial y}+y^2\frac{\partial^2 f}{\partial y^2}=6f.$$

5. a) If $u=\sin^{-1}x+\sin^{-1}y$ & $v=x\sqrt{1-y^2}+y\sqrt{1-x^2}$ then show that u & v are functionally Related. Also find the relationship between them. **8**

b) Divide 24 into three parts so that the continued product of first, square of second & cube of third is maximum. **8**

OR

6. a) The temperature T at any point (x, y, z) in space is given by $T=400xyz^2$. Find the maximum & minimum temperature on the surface of the unit sphere $x^2+y^2+z^2=1$. **8**

b) If $x=r\sin\theta\cos\phi,\ y=r\sin\theta\sin\phi,\ z=r\cos\theta.$ **8**

then show that $\frac{\partial(r,\theta,\phi)}{\partial(x,y,z)}=\frac{1}{r^2\sin\phi}$

7. a) By differentiation under integral sign. Show that. **8**

$$\int_1^0\frac{x^a-x^b}{\log x}\,dx=\log\frac{(a+1)}{(b+1)}$$

b) **4**

Evaluate $\int_0^{\Pi/2}\sqrt{\cot\theta}\,d\theta$

c) Show that $\int_0^1 \frac{dx}{\sqrt{-\log x}} = \sqrt{\Pi}.$ **4**

OR

8. a) Find the root mean square value of $f(x) = e^x + 1$ over the range $x = 0$ to $x = 2$. **8**

b) By differentiating under the integral sign find f(z) if **8**

$$f(\alpha) = \int_0^\infty \frac{e^{-x} - e^{-\alpha x}}{x \sec x} dx$$

9. a) Use the method of least squares to fit a curve $y = ax + bx^2$ to following data. **8**

X	-2	-1	0	1	2
Y	-72	-46	-12	35	93

Calculate the value of y when
x = 1.2.

b) Two lines of regression are given by $x + 2y - 5 = 0$ & $2x + 3y - 8 = 0$. **8**

If $6_x^2 = 12$ find

i) The mean value of x & y.
ii) The standard deviation of y.
iii) The co-efficient of correlation between x & y.

OR

10. a) Calculate the rank correlation. Coefficient for the following data. **8**

x	46	64	66	42	43	22	40	45	50	40	34	60
y	57	41	72	34	30	12	33	45	40	33	32	75

b) Fit a curve $y = ax^b$ to the following data. **8**

x	1	2	3	4	5	6
y	2.98	4.26	5.21	6.10	6.80	7.50

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