M.Sc.-II (Mathematics) (CBCS Pattern) Fourth Semester CBCS

PSCMTHT20.3-Optional Paper-XX - Coding Theory

P. Pages : 2 **GUG/W/18/11405**

Time : Three Hours * 3 4 9 8 * Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Prove that : **10**
If x, y, z be words of length n over A.
Then
i) $0 \le d(x, y) \le n$ ii) $d(x, y) = 0$ if and only if $x = y$
iii) $d(x, y) = d(y, x)$ iv) $d(x, y) \le d(x, y) + d(y, z)$

b) Define : **10**
i) Distance of a code
ii) (n, m, d) - code
Find the distance of the ternary code C = {000000, 000111, 111222}

OR

c) Prove that a code with distance d is exactly (d-1) - error - detecting code. **10**

d) For the ternary code C = {00122, 12201, 20110, 22000}, use the decoding rule to decode the received word 01122. **10**

UNIT - II

2. a) Let V be a vector space over F_q. If **10**
dim(V) = k, then prove that
i) V has q^k elements
ii) V has $\frac{1}{k!}\prod_{i=0}^{k-1}(q^k - q^i)$ different bases.

b) If $x, y \in F_2^n$, then prove that $wt(x+y) = wt(x) + wt(y) - 2\,wt(x * y)$. **10**

OR

c) Let S be a subset of F_q^n, then prove that $\dim(\langle S\rangle) + (S^{\perp}) = n$. **10**

d) Show that – Let C be an [n, k] - linear code over F_q, with generator matrix G. Then VE **10**
F_q^n belong to $C^{\perp}$ if and only if V is orthogonal to very row of G. In particular, given an (n–k) xn matrix H is a parity-check matrix for C if and only if the rows of H are linearly independent and $HG^T = 0$.

UNIT - III

3. a) Show that - let I be a nonzero ideal in $F_q[x]/(x^n-1)$ and let g(x) be a non zero monic polynomial of the least degree in I. Then g(x) is a generator of I and divides x^n-1. **10**

b) Prove that – **10**
Each monic divisor of x^n-1 is the generator polynomial of some cyclic code in F_q^n.

OR

c) Let g(x) be the generator polynomial of a $q-$ary$[n, k]$ - cyclic code C. **10**
Put $h(x)=(x^n-1)/g(x)$. Then show that $h_0^{-1}h_R(x)$ is the generator polynomial of $C^{\perp}$, where h_0 is the constant term of h(x).

d) Let $H=(I_{n-k}\,|\,A)$ be a parity-check matrix of a q-ary cyclic code C. let g(x) be the generator polynomial of C. Then prove that the syndrome of a vector $W \in F_q^n$ is equal to $(W(x)(\mathrm{mod}\, g(x))$. **10**

UNIT - IV

4. a) Prove that Reed-Solomon codes are MCD. **10**

b) Prove that - A narrow - sense binary BCH code of length $n=2^m-1$ as designed distance $\delta=2t+1$ has dimension at least $n-m(\delta-1)/2$. **10**

OR

c) Show that - A BCH code with designed distance δ has minimum distance at least δ. **10**

d) Prove that the polynomials $g_Q(x)$ and $g_N(x)$ belong to $F_\ell(x)$. **10**

5. a) Define : **5**
i) Code Alphabet
ii) q-ary word
iii) q-ary block code
iv) codeword
v) Size of code

b) For the binary linear code Find **5**
i) The number of bases for C.
ii) List all the bases of C.
iii) All generator matrices of C.

c) List all the monic divisor of x^6-1. **5**

d) Define : **5**
i) Primitive BCH code.
ii) Narrow - sense code.

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