M.SC. Part-II (Mathematics) (CBCS Pattern) Fourth Semester CBCS

PSCMTHT18 - Integral Equations Paper-III

P. Pages : 2 **GUG/W/18/11397**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **Five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Find the Solution of integro- differential equation **10**

$$u'(x)+\int_0^1 \exp(x-y)u(y)dy = f(x).\ 0 \le x \le 1 \text{ where } u(0)=0$$

b) Turn the D.E. for Y into an integral equation for f $y''-\lambda y=\cos x;$ with y=0 at x= 0 y'=0 at x=1. **10**

OR

c) Form the integral equation corresponding to $y''(x)+a_1(x)y'(x)+a_2(x)y(x)=f(x)$ with $y(0)=y_0,\ y'(0)=y_1.$ **10**

d) Find an integral equation formulation for the problem defined by **10**

$\frac{d^2y}{dx^2}+4y=f(x),\ 0\le x\le \frac{\pi}{2}$ with the boundary conditions y=0 at x=0 and $y'=0$ at x=$\frac{\pi}{2}$

UNIT – II

2. a) Let k(x, y) be a complex valued continuous function defined on a domains $a\le x\le b, a\le y\le b$ Let f(x) be a complex valued continuous function defined on the integral $a\le x\le b$ then show that the integral equation. $\phi(x)=\lambda\int_a^b k(x,y)\phi(y)dy+f(x)$ has a unique solution. **10**

b) Find the eigen values and eigen functions associated with the integral equal **10**

$$\phi(x)=\lambda\int_0^x k(x,y)\phi(y)dy,\ \text{where } k(x,y)=\begin{cases}\cos x\sin x,\ 0\le x\le y\le\pi\\ \cos y\sin x,\ 0\le y\le x\le\pi\end{cases}$$

OR

c) Find the eigen values and eigen functions of the System defined by **10**

$$\phi(x)=\lambda\int_0^1 (1+xt)\phi(t)dt,\ 0\le x\le 1$$

d) Solve the integral equation. $\phi(x)=\lambda\int_0^1 (1+xt)\phi(t)dt+f(x)$ **10**

UNIT – III

3. a) Solve the integral equation. $\phi(x) = x + 1 + \int_{o}^{x} [1 + 2(x - y)]\phi(y)\,dy$ **10**

b) Find a Fourier series solution for the integral equation. **10**

$$f(x) = \frac{1}{2\pi}\int_{-x}^{x} \frac{1-\alpha^2}{1-2\alpha\cos(x-y)+\alpha^2}\phi(y)\,dy, \quad 0 < \alpha < 1, -\pi \le x \le \pi$$

OR

c) Solve the integral equation. $\phi(x) = 3\int_{0}^{x} \cos(x-y)\phi(y)\,dy + e^{x}$ Using Laplace Transform. **10**

d) Solve the integral $\phi(x) = \lambda\int_{0}^{x} e^{k}(x-y)\phi(y)\,dy + f(x)$ **10**

UNIT – IV

4. a) Find the two term approximation to the solution of the integral equation **10**

$$\phi(x) - \int_{0}^{1} k(x,y)\phi(y)\,dy = x, \; 0 \le x \le 1 \text{ where } k(x,y) = x \;\; x \le y, \; = y \; x \ge y$$

using Galerkin's method.

b) Find the first three functions in the iterative solution of I.E. $\phi(x) = \lambda\int_{0}^{1} \sin xy\,\phi(y)\,dy + 1$ **10**

OR

c) Solve the integral equation. $\int_{0}^{1} \frac{h(u)}{u-w}\,du = 1, 0 \le w \le 1$ **10**

d) Calculate an approximation of the form $a_0 + a_1 y$ to $\phi(y)$ is given by the integral equation. **10**

$$\int_{0}^{1} e^{xy}\phi(y)\,dy = (x+1)^{-1}\left[e^{x+1} - 1\right], 0 \le x \le 1$$

5. a) Show that the integro differential equation $e^{2x} = \int_{0}^{\pi} \sin(x+y)\phi(y)\,dy, 0 \le x \le \pi$ **5**

is not self consistent and so does not have a solution.

b) If the eigen value exists prove that they are real. **5**

c) Solve the integral equation $\int_{0}^{x} \sin\alpha(x-y)\phi(y)\,dy = 1 - \cos\beta x$ **5**

d) If $f(t) \in p^1(R)$ and $f(t) \in A(R)$, where R is a whole Real line then prove that **5**

$$\int_{-\infty}^{\infty} f(x+t)\frac{\sin(\lambda t)}{t}\,dt \to \pi/2 \cdot \left[f(x^{+}) - f(x^{-})\right]$$

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