M.Sc.(Mathematics) (CBCS Pattern) Third Semester

PSCMTHT15-5 - (Optional) Paper-XV: Fuzzy Mathematics-I

P. Pages : 3 **GUG/W/18/11290**

Time : Three Hours Max. Marks : 100

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Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Prove that a fuzzy set A on R is convex if and only if **10**
$A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)$.

b) Let A, B $\in \mathcal{F}$ (X). Then prove that for all $\alpha, \beta \in [0, 1]$. **10**

i) ${}^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$ and ${}^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$

ii) ${}^{\alpha}(\overline{A}) = {}^{(1-\alpha)+}\overline{A}$.

OR

c) Let $A_i \in \mathcal{F}$ (X) for all $i \in I$, where I is an index set. Then prove that **10**

$$\bigcup_{i\in I} {}^{\alpha+}A_i = {}^{\alpha+}\left(\bigcup_{i\in I} A_i\right) \text{ and } \bigcap_{i\in I} {}^{\alpha+}A_i \subseteq {}^{\alpha+}\left(\bigcap_{i\in I} A_i\right)$$

d) Let C be a function from [0, 1] to [0, 1]. Then prove that C is a fuzzy complement iff there exists a continuous function f from [0, 1] to R such that f (1) = 0, f is strictly decreasing and $c(a) = f^{-1}(f(0) - f(a))$ for all $a \in [0, 1]$. **10**

UNIT - II

2. a) Let MIN and MAX be binary operations on R defined by. **10**

$$\text{MIN}(A, B)(z) = \sup_{z=\min(x,y)} \min[A(x), B(y)]$$

and

$$\text{MAX}(A, B)(z) = \sup_{z=\max(x,y)} \min[A(x), B(y)]$$

for all $z \in R$
respectively. Then prove that for any A, B, C $\in$ R.
$\text{MIN}[A, \text{MAX}(B, C)] = \text{MAX}[\text{MIN}(A,B), \text{MIN}(A,C)]$.

b) Let $A \in \mathcal{F}$ (R) then prove that A is fuzzy number iff there exists a closed interval $[a, b] \neq \phi$ such that. **10**

$$A(x) = \begin{cases} 1 & \text{for } x \in [a,b] \\ \ell(x) & \text{for } x \in (-\infty, a) \\ r(x) & \text{for } x \in (b, \infty) \end{cases}$$

where ℓ is a function from $(-\infty, a)$ to [0, 1] that is monotonic increasing, continuous from the right, and such that $\ell(x) = 0$ for $x \in (-\infty, w_1)$; r is a function from (b, ∞) to [0, 1] that is monotonic decreasing, continuous from the left, and such that $r(x) = 0$ for $x \in (w_2, \infty)$.

OR

c) Consider two triangular-shape fuzzy numbers A and B defined as: **10**

$$A(x) = \begin{cases} 0 & \text{for} \quad x \le -1 \text{ and } x > 3 \\ (x+1)/2 & \text{for} \quad -1 < x \le 1 \\ (3-x)/2 & \text{for} \quad 1 < x \le 3 \end{cases}$$

$$B(x) = \begin{cases} 0 & \text{for} \quad x \le 1 \text{ and } x > 5 \\ (x-1)/2 & \text{for} \quad 1 < x \le 3 \\ (5-x)/2 & \text{for} \quad 3 < x \le 5 \end{cases}$$

and their $\alpha-$cuts are

$^{\alpha}A = [2\alpha - 1,\ 3 - 2\alpha]$

$^{\alpha}B = [2\alpha + 1,\ 5 - 2\alpha]$

then find the values of

$^{\alpha}(A+B)$, $^{\alpha}(A-B)$, $^{\alpha}(A \cdot B)$, $^{\alpha}(A/B)$, $A+B(x)$, $(A-B)(x)$, $(A \cdot B)(x)$ and (A/B) x.

d) Find the solution for the equation $A \cdot X = B$ where A and B are triangular-shape fuzzy numbers given by. **10**

$$A = \begin{cases} 0 & \text{for} \quad x \le 3 \text{ and } x > 5 \\ x-3 & \text{for} \quad 3 < x \le 4 \\ 5-x & \text{for} \quad 4 < x \le 5 \end{cases}$$

$$B = \begin{cases} 0 & \text{for} \quad x \le 12 \ \&\ x > 32 \\ (x-12)/8 & \text{for} \quad 12 < x \le 20 \\ (32-x)/12 & \text{for} \quad 20 < x \le 32 \end{cases}$$

UNIT - III

3. a) Determine the transitive max-min closure $R_T(x, X)$ for a fuzzy relation R (x, X) defined by the membership matrix. **10**

$$R = \begin{bmatrix} .7 & .5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & .4 & 0 & 0 \\ 0 & 0 & .8 & 0 \end{bmatrix}$$

b) For any fuzzy relation R on X^2. Prove that the fuzzy relation $R_{T(i)} = \bigcup_{n=1}^{\infty} R^{(n)}$ is the i-transitive closure of R. **10**

OR

c) Prove that for any $a, a_j, b, d \in [0, 1]$, where j takes values from an index set J, operation w_i has the following properties. **10**

i) $i(a,b) \le d$ iff $w_i(a,d) \ge b$

ii) $w_i[i(a, b), d] = w_i[a, w_i(b, d)]$

iii) $w_i\left[\sup_{j \in J} a_j, b\right] = \inf_{j \in J} w_i(a_j, b)$.

d) Let R be a reflexive fuzzy relation on X^2, where $|X| = n \geq 2$. Then prove that $R_{T(i)} = R^{n-1}$ **10**

UNIT - IV

4. a) Let $Q = \begin{bmatrix} .1 & .4 & .5 & .1 \\ .9 & .7 & .2 & 0 \\ .8 & 1 & .5 & 0 \\ .1 & .3 & .6 & 0 \end{bmatrix}$ and $r = [.8 \quad .7 \quad .5 \quad 0]$, determine all solutions of $p \cdot Q = r$, where **10**
$p = [p_j / j \in J]$, $Q = [q_{jk} / j \in J,\ k \in K]$ $r = [r_k / k \in K]$.

b) Let the t-norm i employed in $P \overset{i}{\circ} Q = R$ be the product and let **10**
$Q = \begin{bmatrix} .1 \\ .2 \\ .3 \end{bmatrix}$ and $R = \begin{bmatrix} .12 \\ .18 \\ .27 \end{bmatrix}$ then find the greatest solution P.

OR

c) If $S(Q, R) \neq \phi$ for $P \overset{i}{\circ} Q = R_1$ then prove that $P = \left(Q \overset{w_i}{\circ} R^{-1}\right)^{-1}$ is the greatest member of $S(Q, R)$. **10**

d) Prove that $P = \left(Q \overset{w_i}{\circ} R^{-1}\right)^{-1}$ is the greatest approximation of $P \overset{i}{\circ} Q = R$. **10**

5. a) For any $A \in \mathcal{F}(X)$ then prove that $\alpha_A = \bigcap_{\beta<\alpha} {}^{\beta}A = \bigcap_{\beta<\alpha} \beta^{+}A$. **5**

b) Let MIN be binary operation on R defined by **5**
$\text{MIN}(A, B)(z) = \sup_{z=\min(x,y)} \min[A(x), B(y)]$ then for any $Z, A, B, C \in R$ prove that
$\text{MIN}[\text{MIN}(A, B), C] = \text{MIN}[A, \text{MIN}(B, C)]$.

c) Write the basic properties of the sup-i composition under the standard fuzzy union and intersection. **5**

d) Explain an approximate solutions of fuzzy relations. **5**

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