M.Sc.(Mathematics) (CBCS Pattern) Third Semester

PSCMTHT14-3 - Paper-XIV (Optional) : Graph Theory

P. Pages : 2 | GUG/W/18/11284

Time : Three Hours | Max. Marks : 100

3387

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Prove that if G be a non-empty graph with at least two vertices. Then G is bipartite if and only if it has no odd cycles. **10**

b) Prove that in a graph G there is an even number of odd vertices. **10**

OR

c) Prove that for a tree T with n-vertices then it has precisely n-1 edges. **10**

d) Prove that A graph G is connected if and only if it has a spanning tree. **10**

UNIT - II

2. a) In Dijkstra's algorithm, if at some stage $\lambda(v)$ is finite for the vertex V then. Prove that there is a path from s to v whose length is $\lambda(v)$. **10**

b) Let G be a graph with n vertices, where $n \geq 2$. then prove that G has at least two vertices which are not cut vertices. **10**

OR

c) Prove that: A connected graph G is Euler if and only if the degree of every vertex is even. **10**

d) Prove that Fleury's algorithm produces an Euler tour in an Euler graph G. **10**

UNIT - III

3. a) Let G be a plane graph with n vertices, e edges, f faces and k connected components, then prove that n-e +f = k + 1. **10**

b) State and prove Euler's formula. **10**

OR

c) Show that if a planar graph G of order n and size m has r regions and k components, then n - m + r = k + 1. **10**

d) Let G be a connected plane graph with n vertices, e edges and f faces. Let n*, e* and f* denote the number of vertices, edges and faces respectively of G* then prove that n* = f, e* = e, and f* = n **10**

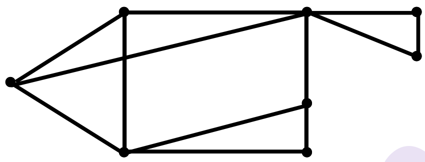
UNIT - IV

4. a) Prove that if D be a weakly connected digraph with at least two vertices. Then D has a directed Euler trail if and only if D has two vertices u and v such that **10**
od(u) = id(u) + 1 and id (v) = od(v) + 1
and, for all other vertices W of D, od(w) = id(w), furthermore, in this case the trail begins at u and ends at v.

b) Prove that : every tournament T has a directed Hamiltonian path. **10**

OR

c) Find the orientation of the graph. **10**



d) Let u and v be two distinct vertices of the graph G. Prove that **10**
i) A set S of vertices of G is u-v separating if and only if every u-v path has at least one internal vertex belonging to S.
ii) A set F of edges of G is u-v separating if and only if every u-v path has at least one edge belonging to F.

5. a) Let G be a graph with n vertices and e edges and let m be the smallest positive integer such that $m \; Z \; \frac{2e}{n}$ -Prove that G has a Vertex of degree at least m. **5**

b) Write down the steps involved in Dijkstra's algorithm. **5**

c) Let G_1 and G_2 be two plane graphs which are both redrawing's of the same planar graph G. Then prove that $f\left(G_1\right) = f\left(G_2\right)$. **5**

d) Prove that a simple graph G is Hamiltonian if and only if its closure C(G) is Hamiltonian. **5**

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