M.Sc. - II (Mathematics) (CBCS Pattern) Third Semester

PSCMTHT13 - PAPER-XIII : Mathematical Methods

P. Pages : 2 **GUG/W/18/11280**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all the **five** questions.
2. Each question carry equal marks.

UNIT – I

1. a) Show that $F_c\left[e^{-a^2 t}, \xi\right] = \frac{1}{\sqrt{2}}\,\frac{1}{a}\,e^{-\xi^2/4a^2}$ **10**

b) Solve the equation. **10**

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with the conditions.

i) $u(0,t) = 0$

ii) $u(x,0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 0 \end{cases}$

OR

c) If $f(t) \in p[a,b]$ where $0 < a < b < \infty$ then prove that **10**

$\int_a^b f(t)\, \mathrm{sun}\lambda t\, dt \to 0$ as $\lambda \to \infty$.

d) Find the Fourier sine & cosine transform of e^{-at}. **10**

UNIT – II

2. a) Solve by Laplace transform method **10**

$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 3y = e^{-x}\sin x,$

with $y(0) = 0,\ y'(0) = 1$.

b) State & prove the convolution theorem. **10**

OR

c) If $L[f(t)] = \bar{F}(p)$ then show that $L\left[t^n F(t)\right] = (-1)^n \frac{d^n}{dp^n}\,\bar{F}(p),\ n = 1, 2, ---$ **10**

d) Solve: $X''(t) + 2x'(t) + x(t) = 5e^t, x(0) = 1, x'(0) = 0$. **10**

UNIT – III

3. a) Show that: **10**

$$f_s[x(a-x),n]=\frac{2a^3}{n^3\pi^3}\left[1+(-1)^{n+1}\right].$$

b) Obtain the finite Fourier sine transform of cosax & sinax. **10**

OR

c) By using generalised transform method obtain the solution $u(x,t)$ of diffusion equation **10**

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{k}\frac{\partial u}{\partial t},\ 0\le x\le 1,\ t\ge 0$$

Satisfying the boundary conditions $u(0,t)=0,\ t>0,$

$\frac{\partial u}{\partial x}(1,t)+hu(1,t)=0,\ t>0$ & initial condition $u(x,0)=f(x),\ 0\le x<1.$

d) Define Fourier sine transform & obtain $F_s[1;n]$. **10**

UNIT – IV

4. a) Define Mellin transform. **10**

Obtain :

i) $M\left[e^{-\alpha x}\right],\alpha>0$ ii) $M[F(ax)].$

b) Show that $h_1,\nu\left[x^{\nu};n\right]=\frac{a^{\nu+1}}{\xi_n}J_{\nu+1}(\xi n\cdot a)$ **10**

OR

c) Find Hankel transform of **10**

$$\begin{aligned} f(x)&=a^2-x^2,\ 0<x<a,\ n=0\\ &=0,\qquad x>a,\ n=0\end{aligned}$$

d) Obtain = $H_\nu\left[x^{\nu-1},\xi\right]$. **10**

UNIT – V

5. a) Define Fourier sine & cosine transform. **5**

b) Find : $L^{-1}\left[\frac{p-1}{p^2-6p+25}\right]$. **5**

c) Find finite Fourier sine transform of e^{ax} . **5**

d) Obtain the Hankel transform of order zero of $\frac{1}{x}$. **5**

****