M.Sc.(Physics) (CBCS Pattern) First Semester

PSCPHYT01 (Core-I) Paper-I - Mathematical Physics

P. Pages : 2 **GUG/W/18/11179**

Time : Three Hours * 3 2 8 2 * Max. Marks : 80

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1. Either

a) Define curl of a vector? If V is a vector field then find curl of V in terms of curvilinear coordinates. **8**

b) Find curl and divergence of $\overline{V}$ **8**

$$\overline{V} = \frac{x\hat{i} + yj + 2\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$$

OR

e) State and prove Fourier integral theorem. **8**

f) Find Fourier sine transform of **8**

$$f(x) = \frac{e^{-ax}}{x}$$

2. Either

a) What do you mean by symmetric and antisymmetric tensor? Show that any second order tensor can be expressed as the sum of symmetric and skew symmetric tensors. **8**

b) What are metric tensors? Obtain the components of metric tensor in three dimensional space in terms of spherical polar coordinates. **8**

OR

e) If $(ds)^2 = r^2(d\theta)^2 + r^2 \cdot \sin^2\theta(d\phi)^2$, Find the values of **8**

i) [22, 1] ii) [12, 2]
iii) [1, 22] iv) [2, 12]

f) Find the inverse Laplace transform of **8**

$$\frac{s^2 - 1}{(s^2 + 1)^2}$$

3. Either

a) Find the eigen values and eigen vectors of the matrix **8**

$$A = \begin{bmatrix} b & a & a \\ a & b & a \\ a & a & b \end{bmatrix}$$

b) Show that all eigen values of a Hermitian matrix are real. **8**

OR

e) State and prove Cayley-Hamilton theorem. **8**

f) Explain. **4**
i) Vector space.
ii) Inner product space.

g) Diagonalise the matrix **4**

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

4. Either

a) Solve the differential equation by power series solution **8**

$$2x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - (x+1)y = 0$$

b) Express $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre polynomials. **8**

OR

e) Prove that for Bessel's function $J_n(x)$ **8**

$$J-n(x) = (-1)^n J_n(x)$$

Where n is positive integer.

f) Prove that recursion formula $xJ'_n = nJ_n - xJ_{n+1}$. **8**

5. Attempt all the followings.

a) What are Dirichlet's condition for Fourier series. **4**

b) What are Christoffel symbols? Obtain the transformation law for Christoffel symbol of first and second kind. **4**

c) Prove that the matrix **4**

$$\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$ is unitary.

d) What is generating function of Bessel function. Show that **4**

$$2J_n' = J_{n-1} - J_{n+1}$$

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