101 - Applied Mathematics-II

P. Pages : 2 **GUG/W/16/3672**

Time : Three Hours Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of non programmable calculator is permitted.

1. a) Solve $\frac{dy}{dx} + \frac{1}{x}\tan y = \frac{1}{x^2}\tan y \sin y$ **5**

b) Solve : $\left[\cos x \log(2y-8) + \frac{1}{x}\right] dx + \frac{\sin x}{y-4} dy = 0$ **4**

c) Solve : $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y\cos(e^x)$ **7**

OR

2. a) Solve : $\frac{dy}{dx} = \frac{y+1}{(y+2)e^y - x}$ **5**

b) Solve : $(\sec x \tan x + \tan y - e^x)dx + \sec x \sec^2 y\, dy = 0$ **4**

c) Solve by variation of parameters $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}\log x$ **7**

3. a) Solve : $\frac{d^2y}{dx^2} + 4y = x\sin x + \cos^2 x$ **8**

b) Solve : $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x + \log x$ **8**

OR

4. a) Solve : $\frac{d^2y}{dx^2} = 2(y^3 + y)$ under the condition $y = 0, \frac{dy}{dx} = 1$ when x = 0. **8**

b) Solve : $\frac{dx}{dt} + 3x - 2y = 1$ **8**

$\frac{dy}{dt} - 2x + 3y = e^t$

given x = y = 0 at t = 0.

5. a) Evaluate $\int_0^{\pi/2}\int_0^{a\sin\theta}\int_0^{\left(\frac{a^2-r^2}{a}\right)} r\,dr\,d\theta\,dz$ **8**

b) Evaluate by changing the order of integration $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}\,dy\,dx$ **8**

OR

6. a) Find the centre of gravity of the area bounded by parabola $y^2 = x$ and line x + y = 2. **8**

b) Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes y + z = 4 and z = 0. **8**

7. a) A particle moves along the curve $x=e^{-t}$, $y=2\cos 3t,\ z=2\sin 3t$ where t is time. Determine its velocity and acceleration vectors. Also the magnitudes of velocity and acceleration at t = 0. **5**

b) Find the directional derivative of $\phi = x^2yz+4xz^2$ at $(1, 2, -1)$ in the direction of $2i-j-2k$. **6**

c) Find the angle between the tangents to the curve $\vec{r}=t^2\hat{i}-2t\hat{j}+t^3\hat{k}$ at points t=1 and t=2. **5**

OR

8. a) The position vector of a point at time 't' is given by $\vec{r}=e^t\cos t\,\hat{i}+e^t\sin t\,\hat{j}$ then show that $\vec{a}=2(\bar{v}-\bar{r})$. **4**

b) Find the value of a, b, c so that the directional derivative of $\phi = axy^2+byz+cz^2x^3$ at point $(1, 2, -1)$ has a maximum magnitude 64 in the direction parallel to z axis. **8**

c) If $\vec{a}=t^2\hat{i}-t\hat{j}+(2t+1)\hat{k},\ \vec{b}=2t\hat{i}+\hat{j}-t\hat{k}$ then find $\frac{d}{dt}(\vec{a}\times\vec{b})$. **4**

9. a) Find the value of n for which vector field $r^n\bar{r}$ is solenoidal. Also show that $r^n\bar{r}$ is conservative vector field. **8**

b) If $\vec{F}=(y\sin z-\sin x)\hat{i}+(x\sin z+2yz)\hat{j}+(xy\cos z+y^2)\hat{k}$ then show that $\vec{F}$ is a irrotional. Hence find its scalar potential. **8**

OR

10. a) Evaluate by Gauss Divergence theorem $\iint (x\hat{i}-y\hat{j}+z^2\hat{k})\cdot\hat{n}\,dS$ where S is the closed surface bounded by the cone $x^2+y^2=z^2$ and the plane z = 1. **8**

b) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ by Stoke's theorem where $\vec{F}=y^2\hat{i}+x^2\hat{j}-(x+z)\hat{k}$ where C is the boundary of the triangles with the vertices (0, 0, 0), (1, 0, 0) and (1, 1, 0). **8**

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