B.E.(with Credits)-Regular-Semester 2012- All Branches Sem I & II

105 - Applied Mathematics -I

P. Pages : 2 **GUG/W/16/3665**

Time : Three Hours Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of nonprogrammable calculator is permitted.

1. a) If $y = 2\tan^{-1} x$ then show that $y_n = 2(-1)^{n-1}(n-1)!\ \sin^n \theta \sin(n\theta)$ where $\theta = \tan^{-1}\left(\frac{1}{x}\right)$. **8**

b) If $\cos^{-1}\left(\frac{y}{b}\right) \log\left(\frac{x}{b}\right)^n$ then show that $x^2 y_{n+2} + (2n+1)x\, y_{n+1} + (n^2+1)y_n = 0$. **8**

OR

2. a) If $y = \sin(m \sin^{-1} x)$ then show that $(1-x^2)\, y_{n+2} - (2n+1)x\, y_{n+1} - (n^2 - m^2)y_n = 0$ **8**
Hence find $y_n(0)$.

b) Show that $\log(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192}$...... **5**

c) Evaluate $\lim_{x\to 0} \frac{x^x - x}{1 - x + \log x}$. **3**

3. a) If $a^2x^2 + b^2y^2 = c^2 z^2$ then show that $\frac{1}{a^2}\frac{\partial^2 z}{\partial x^2} + \frac{1}{b^2}\frac{\partial^2 z}{\partial y^2} = \frac{1}{c^2 z}$. **6**

b) If $f(x, y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log y - \log x}{x^2 + y^2}$ then show that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + 2f = 0$. **4**

c) If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{zx}\right)$ then show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$. **6**

OR

4. a) If $u = \operatorname{cosec}^{-1}\left(\frac{\sqrt{x}+\sqrt{y}}{x+y}\right)$ then show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4\cos 3u}$. **8**

b) If $u = x^2 + y^2, v = 2xy$ then show that $x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} = 2\sqrt{u^2 - v^2}\frac{\partial v}{\partial u}$. **8**

5. a) If $u = x + y + z,\ uv = y + z, uvw = z$ then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = \frac{1}{u^2 v}$. **8**

b) Divide 24 into three parts so that the continued product of first, square of 2^{nd} and cube of third is maximum. **8**

OR

6. a) If $u = \frac{x}{y-z}, v = \frac{y}{z-x}\ w = \frac{z}{x-y}$ then show that u, v, w are functionally related Also find the relation between them. **8**

b) Expand $e^x \log(1+y)$ in powers of x and y by Taylor's series as far as third degree terms. **8**

7. a) By differentiation under integral sign show that $\int_1^0 \frac{x^a - x^b}{\log x} dx = \log\frac{(a+1)}{(b+1)}$. **8**

b) Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$. **4**

c) Show that $\int_0^1 \frac{dx}{\sqrt{-\log x}} = \sqrt{\pi}$. **4**

OR

8. a) Trace the curve $y^2(2a-x) = x^3$. **8**

b) Find the root mean square value of $f(x) = e^x + 1$ over the range x = 0 to x = 2 substitute value of e^2 and e^4 in the result and evaluate it further. **8**

9. a) Find the coefficient of correlation and the lines of regression to the following data. **8**

x:	53	54	55	56	57	58	60	55	46	50
y:	18	18	17	15	14	12	10	9	7	6

b) Fit a least squares parabola $y = ax^2 + bx$ to the following data. **8**

x:	1	2	3	4	5	6
y:	1.8	2.4	3.7	5.2	6.1	8.3

OR

10. a) Find the coefficient of correlation by rank to the following data. **8**

x:	27	31	21	29	20	28	27	30	27	25
y:	112	118	115	111	113	115	110	120	118	114

b) The values of pressure P and volume V at different temperatures are as follows. **8**

V :	1	3	5	7	9	11
P :	103.7	89.2	61.5	38.1	19.6	9.8

Fit a curve $PV^k = C$.

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