ASH3019 - Applied Mathematics - III

P. Pages : 3 **GUG/W/16/3789**

Time : Three Hours Max. Marks : 80

* 3 6 2 9 *

Notes : 1. All questions carry equal marks.
2. Use of nonprogrammable calculator is permitted.

1. a) Find the Laplace transform of $t\,(t+2)\sin t$. **4**

b) Find $L\left\{\dfrac{\cos at - \cos bt}{t}\right\}$ Hence evaluate $\displaystyle\int_0^\infty \frac{\cos at - \cos bt}{t}\,dt$ **5**

c) Express $f(t) = \begin{cases} \sin t & ; 0 < t < \pi \\ \sin 2t & ; \pi < t < 2\pi \\ \sin 3t & ; t > 2\pi \end{cases}$ i **7**

n terms of unit terms of function & hence find it's LT.

OR

2. a) Find $L^{-1}\left[\dfrac{1}{S^3\,(S^2+1)}\right]$ by convolution theorem. **8**

b) Solve by Laplace transform $y''' - 3y'' + 3y' - y = t^2 e^t$ with $y(0) = 1,\ y'(0) = 0\ y''(0) = 2$. **8**

3. a) Find the inverse matrix of $\begin{bmatrix} 4 & -5 & 6 \\ -1 & 2 & 3 \\ -2 & 4 & 7 \end{bmatrix}$ by partitioning method **8**

b) Find the matrix which diagonalizes the matrix A, where $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ **8**

Also write the diagonal matrix.

OR

4. a) Determine the value of α, β, η when $\begin{bmatrix} 0 & 2\beta & \eta \\ \alpha & \beta & -\eta \\ \alpha & -\beta & \eta \end{bmatrix}$ is orthogonal. **4**

b) Test for linear dependency of the vectors & find the relation between them If Possible **5**
$x_1 = (1, 2, 2, 3),\ x_2 = (-1, 0, 3, 1),\ x_3 = (-2, -1, 1, 0)\ x_4 = (-3, 0, -1, 3)$

c) Investigate values of λ & μ so that the system of equations $2x+3y+5z=9,\ 7x+3y-2z=8,\ 2x+3y+\lambda z=\mu$ have **7**

i) No solution

ii) Unique solution

iii) Infinite solution

5. a) Verify Cayley-Hamilton theorem for matrix. **8**

$$A=\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} \text{ and find } A^{-1}$$

b) By Sylvester's theorem, show that **8**

$$e^A = e^x \begin{bmatrix} \cos h\, x & \sin h\, x \\ \sin h\, x & \cos h\, x \end{bmatrix} \text{ where } A=\begin{bmatrix} x & x \\ x & x \end{bmatrix}$$

OR

6. a) Solve the differential equation by matrix method **8**

$$\frac{d^2y}{dx^2} + 4y = 0\ ,\ y(0)=1,\ y'(0)=1$$

b) Reduce the quadratic form $8x^2+7y^2+3z^2-12xy+4xz-8yz$ to the canonical form. **8**

7. Solve

a) $p+3q=5z+\tan(y-3x)$ **4**

b) $(x^2+y^2+yz)\dfrac{\partial z}{\partial x}+(x^2+y^2-zx)\dfrac{\partial z}{\partial y}=z(x-y)$ **6**

c) $(D^2+3DD'+2D'^2)\ z = 24xy$ **6**

OR

8. a) Solve $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial x\,\partial y}-2\dfrac{\partial^2 z}{\partial y^2}=(y+1)e^x-\cos x\ \cos 2y$ **8**

b) Solve $\dfrac{\partial u}{\partial t}=2\dfrac{\partial u}{\partial x}+u$, given that $u(x,0)=3e^{-4x}$ by using method of separation of variables. **8**

9. a) Obtain Fourier series for function $f(x)=\begin{cases} 0 & ;\ -\pi \le x < 0 \\ \sin x & ;\ 0 \le x \le \pi \end{cases}$ Hence show that **8**

$$f(x)=\frac{1}{\pi}-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos 2nx}{4n^2-1}+\frac{1}{2}\sin x$$

b) Obtain half range sine series for function **8**

$$f(x) = \begin{cases} \sin x & ; 0 < x < \pi/4 \\ \cos x & ; \pi/4 < x < \pi/2 \end{cases}$$

OR

10. a) Find the Fourier transform of $f(x) = \begin{cases} 1 & , |x| \le 1 \\ 0 & , |x| > 1 \end{cases}$ **8**

Hence evaluate $\int_0^{\infty} \frac{\sin x}{x}\, dx$

b) Using Fourier integrals show that **8**

$$\int_0^{\infty} \frac{\sin \pi\lambda \, \sin x\lambda}{1-\lambda^2}\, d\lambda = \begin{cases} \frac{\pi}{2}\sin x & 0 \le x \le \pi \\ 0 & , x > \pi \end{cases}$$

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