B.E.(with Credits)-Regular-Semester 2012-Computer Science and Engineering / Computer Technology / Information Technology Sem III

ASH3012 / ASH 3013 / ASH 3017 : Applied Mathematics-III

P. Pages : 3 **GUG/W/16/3694**

Time : Three Hours * 3 5 9 9 * Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) Find the z-Transform of sin (3n + 5). **5**

b) If $z\{f(n)\} = f(z)$ then show that $z\{a^n f(n)\} = f\left(z/a\right)$. **4**

c) Using z-transform method to solve $y_{n+2} + 5y_{n+1} + 6y_n = 6^n$ given that $y_0 = 0,\ y_1 = 1$. **7**

OR

2. a) Using Fourier integral representation show that, **8**

$$\int_0^\infty \frac{\sin \pi\lambda \sin \lambda x}{1-\alpha^2}\, d\lambda = \begin{cases} \pi/2 \sin x & o \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$$

b) Find the Fourier transform of **8**

$$f(x) = \begin{cases} 1 & , |x| \le 1 \\ 0 & |x| > 1 \end{cases}$$

hence find $\int_0^\infty \frac{\sin x}{x}\, dx$

3. a) Find the inverse of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ by partitioning **8**

b) Investigate the value of a and b such that the equations. **8**

$$x + y + 2z = 2$$
$$2x - y + 3z = 2$$
$$5x - y + az = b$$

have :

i) No solution ii) Unique solution iii) Infinite solution

OR

4. a) Investigate the linear dependence of the vectors **5**

$x_1 = (1, 2, 4),\ x_2 = (2, -1, 3)\ x_3 = (0, 1, 2)\ x_4 = (-3, 7, 2)$

b) Find the rank of matrix $A = \begin{bmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{bmatrix}$ **3**

c) Find the modal matrix for the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ **8**

5. a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and hence Find A^{-1}. **8**

b) Use Sylvester's theorem to show that 3 tan A = (tan 3) A where $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ **8**

OR

6. a) Solve by matrix method $\frac{d^2x}{dt} - 4\frac{dx}{dt} + 3x = 0$ where $x(0) = 2,\ x'(0) = 1$ **8**

b) Find the Largest Eigen value and corresponding eigen vector of matrix $A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$ **8**

7. a) Let x be the random variable giving the number of heads in three tosses of a fair coin find **8**
i) The probability function
ii) The distribution function.

b) Let x and y be continuous random variable having joint density function **8**

$$f(x, y) = \begin{cases} C(x^2 + y^2) & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find :
i) Constant C
ii) $p\left(x < \frac{1}{2},\ y > \frac{1}{2}\right)$
iii) Marginal distribution function of x and y.
iv) Determine whether x and y are independent.

OR

8. a) A joint probability function of two random variable 'x' is given by **8**

$$f(x, y) = \begin{cases} cxy & x = 1, 2, 3 \ \&\ y = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

Find i) Constant C
ii) $p(1 < x \le 2,\ y \le 3)$
iii) Marginal density function of x and y
iv) Determine whether x and y are independent

b) A random variable x has density function $f(x) = \frac{c}{x^2+1}$, $-\infty < x < \infty$ **8**

Find i) Constant C

ii) $p\left(\frac{1}{3} \le x^2 \le 1\right)$

iii) Distribution function

9. a) Let $x = \begin{cases} \frac{1}{3} & \text{Prob } \frac{1}{3} \\ -\frac{1}{3} & \text{Prob } \frac{1}{3} \end{cases}$ **8**

Find i) Moments generating function.

ii) First four moment about origin

iii) Variance

b) Let x and y be random variable having joint density function **8**

$$f(x,y) = \begin{cases} e^{-(x+y)} & x \ge 0,\ y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Find i) Var (x)

ii) Var (y)

iii) Con (xy)

iv) ρ

OR

10. a) A random variable x has density function given by **8**

$$f(x) = \begin{cases} 2e^{-2x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

Find i) Moment generating function

ii) First four moment about origin

iii) Variance

b) The joint probability function of two discrete random variable x and y is given by **8**

$$f(x,y) = \begin{cases} \frac{2x+y}{42} & x = 0, 1, 2 \text{ and} \\ & y = 0, 1, 2, 3 \end{cases}$$

Find i) Var (x)

ii) Var (y)

iii) Co-variance between x and y

iv) Coefficient of correlation P.

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