(Hours: 3 hrs)   [Total Marks: 80]

Note: 1. Question no. **1** is compulsory.
         2. Attempt any **three** questions out of remaining **five** questions.

Q.1.[a] Given two lines of regression lines x + 2y - 5 = 0, 2x + 3y + 8 =0.   [5]
Find (i) $\bar{x}$, $\bar{y}$ (ii) correlation coefficient r.

[b] Show that $97 \mid (2^{48} - 1)$.   [5]

[c] The probability density function of a random variable x is zero except at   [5]
x =0, 1, 2 and p(0) = 3k³, p(1) =4k – 10k², p(2) =5k - 1.
Find (i) k (ii) p (0 < x ≤ 2).

[d] Give an example of a graph which has   [5]
(i) Eulerian circuit but not a Hamiltonian circuit
(ii) Hamiltonian circuit but not an Eulerian circuit

Q.2.[a] Find gcd (2947, 3997) using Euclidean Algorithm. Also find x and y   [6]
such that $2947x + 3997y = \gcd(2947, 3997)$.

[b] The four roots of unity 1, -1, i, -i forms a group under multiplication.   [6]

[c] Find whether the following graphs G = (V,E) and G' = (V', E') are   [8]
isomorphic ? Justify.
(1) V = {a,b,c,d} ,E ={(a,b),(a,d), (b,d), (c,d),(c,b),(c,d)}
(2) V' ={1,2,3,4} ,E' = {(1,2),(2,3) ,(3,1),(3,4),(4,1), (4,2)}

Q.3.[a] Show that $(D_8 , \leq)$ is a lattice. Draw its Hasse diagram.   [6]

[b] The local authorities in a certain city install 10,000 electric lamps in the   [6]
streets of the city. If these lamps have an average life of 1000 burning
hours with a standard deviation of 200 hours, how many lamps might be
expected to fail i) in the first 800 burning hours?
                    ii) Between 800 and 1200 burning hours?

[c] **(i)** Find inverse of 2^{-1} (mod 31) using Fermat's theorem.   [8]
(ii) Find the Legendre's symbol of $\left(\frac{19}{23}\right)$.

Q.4.[a] Calculate the coefficient of correlation between x and y from the   [6]
following data

x	12	9	8	10	11	13	7
y	14	8	6	9	11	12	3

[b] (i) Draw a connected graph for which every edge isa cut edge.   [6]
(ii) Show that any connected graph with 'n-1' edges is a tree.

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[c] **(i)** Can it be concluded that the average lifespan of an Indian is more [8] than 70 years if a random sample of l00 Indians has an average lifespan of 71.8 years with standard deviation of 8.9 years?
(ii) Ten individuals are chosen at random from a population and their heights are found to be in inches 63, 63, 64, 65, 66,69, 69, 70, 70,71. Discuss the suggestion that the mean height of the universe is 65 inches.

Q.5.[a] Solve $x \equiv 5 \pmod 6$, $x \equiv 4 \pmod{11}$, $x = 3 \pmod{17}$. [6]
[b] Theory predicts that the proportion of beans in the four groups A, B, C, D [6] should be 9:3:3:1. In an experiment among 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Using Chi-Square verify does the experimental results support the theory?
[c] Let G be a group of all permutations of degree 3 on 3 symbols 1, 2 & 3. [8] Let H ={I, (1 2)} be a subgroup of G. find all the distinct left cosets of H in G and hence index of H.

Q.6.[a] Show that $53^{103} + 103^{53}$ is divisible by 39. [6]
[b] Given L = {1, 2, 4, 5, 10, 20}with divisibility relation. Verify that $(L, \leq)$ [6] is a distributive but not complimented Lattice.
[c] **(i)** Write the following permutation as the product of disjoint cycles [8]
    f = (1 2) (1 2 3) (1 2).
(ii) Express the expression (x +y) (x + z) (x'y)' in the sum-of-product form.

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