

**Total Marks: 80****Hours: 3 hrs**

Note : 1) Question no. 1 is compulsory.

2) Attempt any three questions out of five questions

**Q-1**

a) If any 11 numbers between 1 and 20 are chosen show that at least two of them will be multiples of each other. (05)

b) A function  $f : R - \left\{ \frac{7}{3} \right\} \rightarrow R - \left\{ \frac{4}{3} \right\}$  is defined by  $f(x) = \frac{4x-5}{3x-7}$ , Prove that f is bijective and find the rule for  $f^{-1}$ . (05)c) Find  $L \left[ \frac{d}{dt} \left( \frac{1 - \cos 2t}{t} \right) \right]$  (05)d) Prove that there does not exist an analytic function whose imaginary part is  $3x^2 + \sin x + y^2 + 5y + 4$ . (05)**Q-2**a) Find  $L^{-1} \left[ \frac{s}{(s^2 + 3^2)(s^2 + 5^2)} \right]$  using convolution Theorem. (06)

b) What is the chance of throwing ten with four dice? (06)

c) In a certain examination there are multiple choice questions. There are four possible answers to each questions and one of them is correct. An intelligent student can solve 90% questions correctly by reasoning and for the remaining 10% questions he gives answer by guessing. A week student can solve 20% question correctly by reasoning and for the remaining 80% questions he gives answer by guessing. An intelligent student gets the correct answer. What is the probability that he was guessing. (08)

**Q-3**

a) A can hit a target 2 times in 5 shots, B 3 times in 4 shots, C 2 times in 3 shots. They fire a volley. What is the probability that at least 2 shots hit the target? (06)

b) Find  $L^{-1} \left( \tan^{-1} \left( \frac{2}{s^2} \right) \right)$  (06)c) If R is the relation on the set of integers such that  $aRb$  if and only if  $2a+3b$  is divisible by 5. Find the equivalence classes. (08)**Q-4**a) Evaluate  $\int_{t=0}^{\infty} e^{-3t} \left( \frac{\cos(7t) - \cos(11t)}{t} \right) dt$  (06)b) Find  $L^{-1} \left[ \frac{s^2 + 2s + 3}{(s^2 + 2s + 10)(s^2 + 2s + 17)} \right]$  (06)c) Find the bilinear Transformation which maps the points  $2, i, -2$  on to the points  $1, i, -1$ . Also find image of  $|z| = 1$  of z-plane to w-plane. (08)

Q-5

a) A family consisting of an old man, 6 adults and 4 children is to be seated in a row for dinner. The children wish to occupy two seats at each end and the old man refuse to have a child on either side of him. In how many ways can the seating arrangement be made for the dinner? (06)

b) Find the analytic function  $f(z) = u + iv$  in terms of  $z$  if  $u - v = (x - y)(x^2 + 4xy + y^2)$ . (06)

c) Solve  $\frac{d^3 y}{dt^3} - 2\frac{d^2 y}{dt^2} + 5\frac{dy}{dt} = 0$  with  $y(0) = 0$ ,  $y'(0) = 0$ ,  $y''(0) = 1$ . (08)

Q-6

a) Prove that  $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$  (06)

b) Draw the Hasse diagram of  $D_{105}$ . (06)

c) Find Laplace Transformation of the following

i)  $te^{3t} \operatorname{erf}(\sqrt{t})$ ,

ii)  $\sin t H(t) + (\cos t - \sin t) H(t - \pi)$  (08)