Q. P. Code: 37687

Total Marks: 80 **Time Duration: 3Hr**

N.B.:1) Question no.1 is compulsory.
2) Attempt any three questions from Q.2to Q.6.
3) Figures to the right indicate full marks.

Maximum Marks

Q1. a) Find the Laplace transform of $\cos 2t \sin t\, e^{-t}$. **[5]**

b) Find the half-range sine series for $f(x) = x(\pi - x)$ in $(0, \pi)$. **[5]**

c) Show that the function $f(z) = ze^z$ is analytic and find $f'(z)$ in terms of z. **[5]**

b) Prove that $\nabla\left\{\nabla.\frac{\bar{r}}{r}\right\} = -\frac{2}{r^3}\bar{r}$. **[5]**

Q2. a) Find the inverse Z-transform of $F(z) = \frac{z}{(z-1)(z-2)}$ $|z| > 2$. **[6]**

b) Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$. **[6]**

c) Obtain Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & , -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi} & , 0 \le x \le \pi \end{cases}$, **[8]**

deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

Q3. a) Find $L^{-1}\left[\frac{1}{s^2(s+a)^2}\right]$ using convolution theorem. **[6]**

b) Show that the set of functions $\cos nx, n = 1, 2, 3 \ldots$ is orthogonal on $[0, 2\pi]$. **[6]**

c) Using Green's theorem evaluate $\int_c \left(\frac{1}{y}dx + \frac{1}{x}dy\right)$ where C is the boundary of the region defined by $x = 1, x = 4, y = 1$ and $y = \sqrt{x}$. **[8]**

Q4. a) Find Laplace transform of $f(t) = k\frac{t}{T}$ for 0 < t < T and $f(t) = f(t + T)$. **[6]**

b) Show that $\bar{f} = (x^2 + xy^2)\, i + (y^2 + x^2y)\, j$ is irrotational and find its scalar potential. **[6]**

c) Find half – range cosine series for $f(x) = x$, 0 < x < 2. Using Parseval's identity deduce that **[8]**

i) $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} - \frac{1}{5^4} + \cdots$

ii) $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$

Q5.a) Use divergence theorem to show that $\iint_S \nabla r^2 \overline{ds} = 6v$ where S is any closed surface enclosing a volume V. **[6]**

b) Find the Z-transform of $f(k) = k\alpha^k$, k≥ 0 . **[6]**

c) i) Find $L^{-1}\left[\frac{(s+2)^2}{(s^2+4s+8)^2}\right]$ **[8]**

ii) Find $L^{-1}[2\tanh^{-1} s]$

Q6.a) Solve using Laplace transform **[6]**

$(D^2 - 3D + 2)y = 4e^{2t}$, with $y(0) = -3, y'(0) = 5$.

b) Find the bilinear transformation which maps the points 1, -i, 2 on z-plane onto 0, 2, -i respectively of w-plane. **[6]**

c) Express the function $f(x) = \begin{cases} \sin x , & 0 < x \le \pi \\ 0 & x < 0, x > \pi \end{cases}$ as Fourier integral and deduce that $\int_0^\infty \frac{\cos\left(\frac{w\pi}{2}\right)}{1-w^2} dw = \frac{\pi}{2}$. **[8]**

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