

Max. Marks : 80

(Time: 03 hours)

- N.B. : (1) Question No.1 is **compulsory**
 (2) Attempt **any three** questions from Q.2 to Q.6
 (3) Figures to the right indicate full marks

Q. 1 (a) Find the Laplace Transform of $e^{2t} + 4t^3 - \sin 2t \cos 3t$ 05

(b) Find the Fourier series of $f(x) = x$, $-\pi < x < \pi$ 05

(c) Calculate Spearman's coefficient of rank correlation from the following data 05

X:	12	17	22	27	32
Y:	113	119	117	115	121

(d) Find the constants a, b, c, d, e 05
 if $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is analytic

Q.2 (a) Determine whether the function $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{y}{x}$ is 06
 analytic and if so, find its derivative.

(b) A random variable X has the following probability distribution 06

X	0	1	2	3	4	5	6
P(X=x)	k	3k	5k	7k	9k	11k	13k

Find (i) k, (ii) $P(X < 4)$ (iii) $P(3 < X \leq 6)$

(c) Evaluate $\int_0^\infty e^{-2t} t \cos t dt$ 08

Q.3 (a) Find the Fourier series of $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}$, $-\pi < x < \pi$ 06

(b) A continuous random variable has probability density function 06

$$f(x) = k(x - x^2); \quad 0 \leq x \leq 1$$

Find (i) k, (ii) mean, (iii) variance

(c) Find the inverse Laplace transform of $\frac{s^2 + 2s + 3}{(s^2 + 2s + 5)(s^2 + 2s + 2)}$ 08

Q.4(a) Find the Laplace Transform of $f(t)$, 06

where $f(t) = \cos t$, for $0 < t < \pi$ and $f(t) = \sin t$, for $t > \pi$

(b) Calculate the Karl Pearson's coefficient of correlation from the following data 06

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

- (c) Find the Fourier series of $f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ 2\pi - x, & \pi \leq x \leq 2\pi \end{cases}$ 08

- Q.5 (a) Find the inverse Laplace transform of $\frac{s}{(2s+1)^2}$ 06

- (b) Find the Laplace transform of $t \left(\frac{\sin t}{e^t} \right)^2$ 06

- (c) Find the lines of regression for the following data 08

X:	78	36	98	25	75	82	90	62	65	39
Y:	84	51	91	60	68	62	86	58	53	47

- Q.6 (a) Find the mean and the variance of the following distribution 06
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|--------|-----|-----|-----|-----|
| X | 1 | 3 | 4 | 5 |
| P(X=x) | 0.4 | 0.1 | 0.2 | 0.3 |

- (b) Find the inverse Laplace transform of $\log \left(1 + \frac{a^2}{s^2} \right)$ 06

- (c) Find the analytic function $f(z) = u + iv$ whose imaginary part is 08
- $$v = x^2 - y^2 + \frac{x}{x^2 + y^2}$$